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IDENTIFICATION OF SYSTEM DYNAMICS USING MULTIPLE INTEGRATIONS

William Richard Hansell



United States Naval Postgraduate School



THESIS

IDENTIFICATION OF SYSTEM DYNAMICS
USING MULTIPLE INTEGRATIONS

by

William Richard Hansell

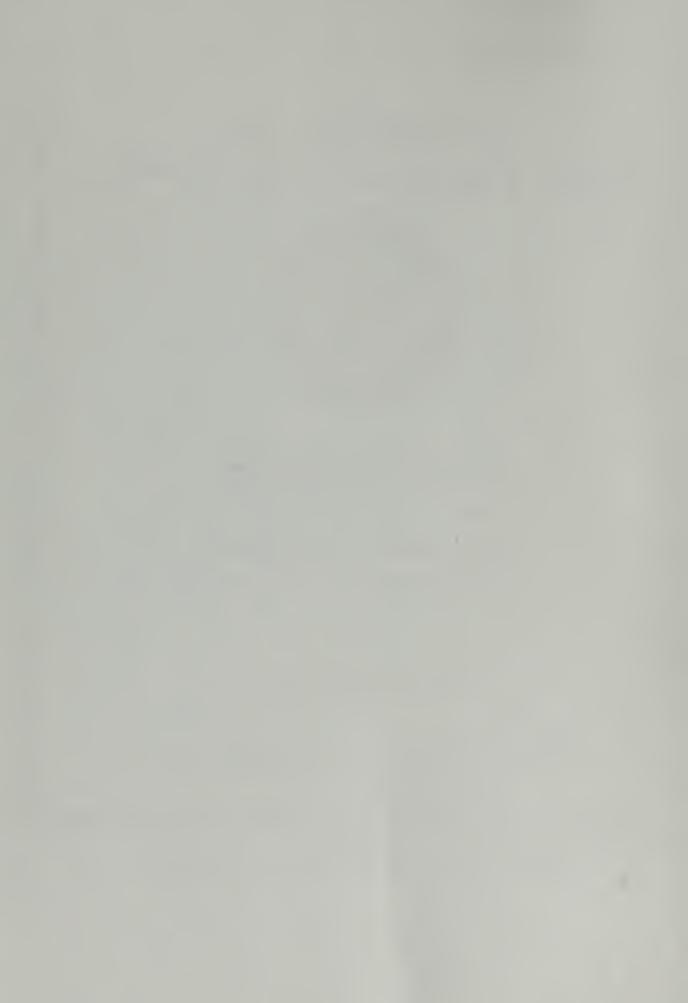
Thesis Advisor:

George J. Thaler

June 1971

T140749

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Identification of System Dynamics Using Multiple Integrations

by

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Submitted in partial fulfillment of the requirements for the degree of

MASTER OF SCIENCE IN ELECTRICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL June 1971



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ABSTRACT

A practical method for identifying linear time invariant systems on the basis of arbitrary input-output records is reviewed and extended to handle the case where the system is not initially in the zero state. The method is implemented using a digital computer program composed of a numerical integration subroutine and a subroutine for solving overdetermined sets of linear algebraic equations. Several examples are presented to demonstrate the accuracy and present capabilities of the procedure.



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ACKNOWLEDGEMENT

The author wishes to express his appreciation to Doctor George J. Thaler of the Electrical Engineering Department of the Naval Postgraduate School for the guidance and helpful suggestions in the preparation of this work.



I. INTRODUCTION

A. THE IDENTIFICATION PROBLEM

In order to apply any of the modern techniques of control system design one must first have a mathematical model of the system to be controlled. The form of this model will depend on the design methods to be employed as well as on the physical characteristics of the system. Since most of the theory on the analysis and design of control systems is based either on the state space or transform representation of systems the vast majority of mathematical models will consist of either a set of state equations or a transfer function.

Once it has been decided what basic form the mathematical model should take the problem of determining the numerical values of the parameters arises. Parameter values can often be determined from the laws of physics and the data supplied by a manufacturer or obtained through testing. This is not always the case however. Occasionally the laws of physics become mathematically intractable or are not even applicable. Quite often the values of certain key parameters are not available. It is in these cases that the identification problem arises.

A common problem in engineering is that of determining the output of a system based on a knowledge of the system model, input, and initial conditions. The identification problem is similar to this but here the unknown quantity is



the system model. The input and output are assumed to be known. For the purposes of this thesis the identification problem can be stated as follows:

Given - a record of the input and output of a system

over some finite period of time,

Find - a mathematical model and the numerical values

of the model parameters in such a way that the

model will accurately describe the behavior

of the system.

It should be kept in mind that the problem of identifying a system solely on the basis of input and output data (the so called "black box" problem) is very rare in engineering. Even in the case where none of the model parameters are known one will more than likely have a fair idea of whether the system is linear or nonlinear, time varying or time invariant, what the order of magnitude of the dominant time constants is, and what types of inputs and outputs are to be expected. For this reason most engineering identification problems fall into the "grey box" category. This distinction may seem trivial but virtually all identification techniques rely heavily on knowledge of the characteristics and quantities mentioned above.

Although the control systems literature on system identification is quite vast there are no known identification schemes capable of handling all identification problems effectively. Choosing a method suitable for a given problem can become a formidable task. A paper summarizing



most of the common approaches to the problem of identifying lumped parameter systems has been published by Nieman, Fisher, and Seborg [1]. A good discussion of the industrial applications of various methods has been published by Eykhoff, et al. [2].

One common approach used in linear system identification is that of obtaining the impulse response of the system. Mishkin and Haddad [3] have developed a technique for finding the impulse response based on samples of the system output and its derivatives. A technique for estimating the impulse response on the basis of noisy input and output samples has been developed by Levin [4] and Kerr and Surber [5]. Turin [6] and Lichtenberger [7,8] have used a matched filter to obtain an identification. The use of a white noise or binary test signal and crosscorrelation has been suggested by Hill and McMurtry [9]. The noise limitations of the sample approximation, matched filter, and crosscorrelation identification techniques have been investigated by Lindenlaub and Cooper [10].

Another common approach to the identification problem is to determine the coefficients of the differential or difference equation which describes the system. Kumar and Sridhar [11] have employed the method of quasilinearization with some success. Nagumo and Noda [12] have developed a learning approach to the problem. Bass [13] has developed a method which uses modulating functions and works well in the presence of noise. Astrom and Bohlin [14] have developed



a statistically optimal method of determining differential equation parameters known as the "maximum likelihood method." A similar method which is not optimal but is considerably simpler computationally has been developed by Peterka and Smuk [15,16]. It is known as the "prior knowledge fitting method." An algorithm for determining state variable models of sampled data systems has been proposed by Ho and Kalman [17]. The algorithm performs quite well in the presence of noise, and has been extended to continuously operating systems by Eldem [18].

Methods of identifying nonlinear and distributed parameter systems are usually limited to specific types of systems or to specific types of nonlinearities. undoubtedly due to the wide variety of nonlinearities encountered in physical systems and the difficulty of finding a model capable of characterizing them all. Shinbrot [19], Mowery [20], Fairman [21], and Bellman, et αl . [22] have all approached the problem of identifying nonlinear systems by assuming a particular form of differential equation is capable of describing the system and then developing methods around the form of differential equation chosen. Another common approach to nonlinear system identification is that of representing a system by a suitable functional polynomial relating the input and output. Hsieh [23] uses this approach and a steepest descent algorithm to solve the identification problem. Similar approaches have been taken by Simpson [24], Bose [25,26], and Hubbell [27].



Identification methods vary widely with respect to how much must be known about the system before the method can be applied. Some identification techniques require that prior estimates of all system parameters be available.

Many methods restrict the allowable system inputs to a family of testing functions such as steps or binary pulses. In general, the less that is known about a given system and the tighter the constraints on the kind of signals which may be applied as inputs the more difficult it is to find a method capable of accomplishing the identification.

B. OBJECTIVES OF THIS PAPER

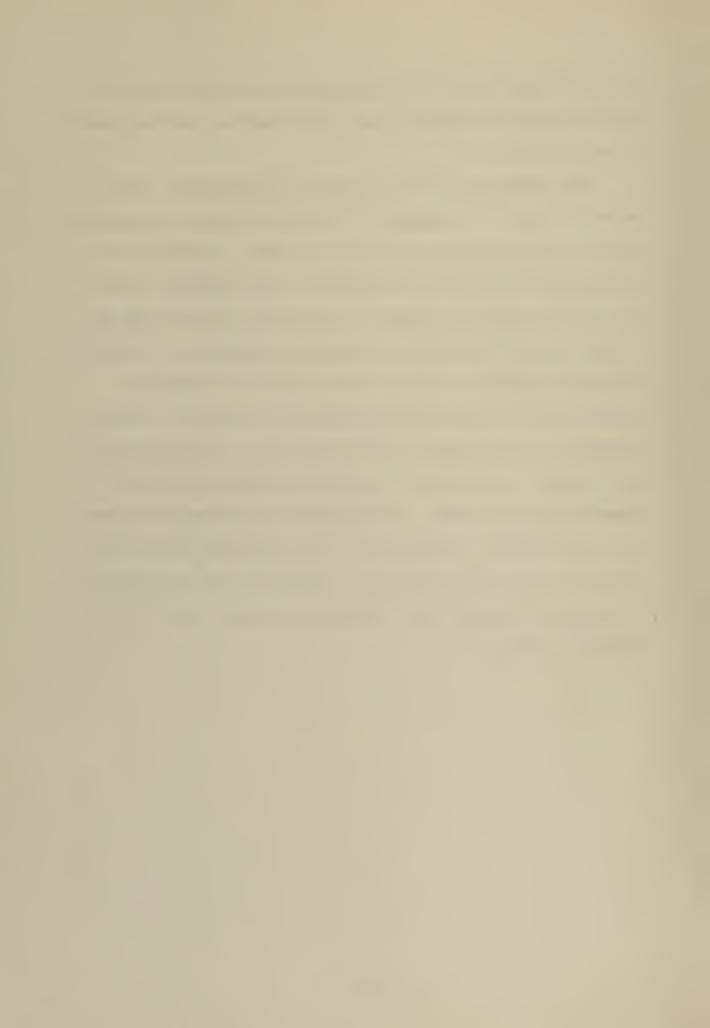
This paper will present a study of an identification technique originally suggested by Diamessis [28]. It is designed to identify lumped linear time invariant systems but has been extended by Diamessis [29] and Wang [30] to handle certain types of nonlinearities. The technique requires a knowledge of the system input and output over some finite time interval as well as a rough estimate of the system order. The system input need not be restricted to a class of testing functions, it can be completely arbitrary.

Unlike some identification techniques which require the calculation of derivatives of the input and output, the technique to be presented requires only integrals of the input and output. The advantages of numerical integration over differentiation are well known. Since any zero mean noise component on the input or the output tends to be



attenuated greatly by the integration process the system identification can be more accurate than the raw data used to accomplish it.

The remainder of this thesis is divided into three major sections. In Chapter II the theoretical development of the identification technique is given. A method for identifying the initial conditions of the unknown system is also presented. Chapter III presents a method for implementing the techniques developed in Chapter II. Particular attention is given to the choice of numerical methods and to efficient programming techniques. Several examples are presented to demonstrate the capabilities of the method. In Chapter IV several recommendations for further study are made. Conclusions concerning the accuracy and present limitations of the technique under consideration are also discussed. Following the conclusions a complete listing of all computer programs used in the thesis is given.



II. IDENTIFICATION BY MULTIPLE INTEGRATIONS

A. GENERAL APPROACH

The development which follows is similar to the development given by Diamessis [28] in 1965. There are a few notable differences however. The development given by Diamessis is restricted to the case where all initial conditions are zero. This is a rather serious restriction since it may be difficult or impossible to find a point where the system is in the zero state if the systems operation is not to be disturbed. Zero initial conditions will not be assumed in the development which follows. A method for solving for the unknown initial conditions will be presented. Diamessis proposed the formulation of a uniquely determined set of linear algebraic equations with the model parameters as unknowns. This development will make use of overdetermined sets of linear algebraic equations with the model parameters and initial conditions as unknowns. The overdetermined set of equations will then be solved using the method of least squares. It will be shown that this results in a more accurate identification when the accuracy of the available data is limited and the order of the system is unknown.

Any single input, single output, lumped parameter,
linear, time-invariant system can be described by a linear
ordinary differential equation with constant coefficients.



The basic form of this equation is given in Equation (1) along with a set of initial conditions.

$$\frac{d^{n}y(t)}{dt^{n}} + a_{n-1} \frac{d^{n-1}y(t)}{dt^{n-1}} + \cdots + a_{0}y(t)$$
 (1)

$$= b_{m} \frac{d^{m}u(t)}{dt^{m}} + \cdots + b_{0}u(t)$$

with initial conditions;

$$\frac{d^{n-1}y(0)}{dt^{n-1}} = y_0^{n-1} \qquad \frac{d^{m-1}u(0)}{dt^{m-1}} = u_0^{m-1}$$

$$\vdots \qquad \vdots \qquad \vdots$$

$$y(0) = y_0 \qquad u(0) = u_0$$

where;

u(t) = system input

y(t) = system output

The identification problem to be treated here consists of determining n, m, and the various coefficients of the differential equation on the basis of input and output records taken over some arbitrary time interval. The initial conditions will be assumed to be unknown.

Taking the Laplace transform of Equation (1) yields Equation (2).

$$s^{n}Y(s) + a_{n-1}s^{n-1}Y(s) + \cdots + a_{o}Y(s)$$
 (2)
 $b_{m}s^{m}U(s) + \cdots + b_{o}U(s)$
 $+ g_{n-1}s^{n-1} + \cdots + g_{o}$



The g_i coefficients account for the contributions of the initial conditions. Dividing Equation (2) by s^{n+1} is equivalent to integrating n+1 times in the time domain.

$$\frac{Y(s)}{s} + a_{n-1} \frac{Y(s)}{s^{2}} + \cdots + a_{0} \frac{Y(s)}{s^{n+1}}$$

$$= b_{m} \frac{U(s)}{s^{n-m+1}} + \cdots + b_{0} \frac{U(s)}{s^{n+1}}$$

$$+ g_{n-1} \frac{1}{s^{2}} + \cdots + g_{0} \frac{1}{s^{n+1}}.$$
(3)

Taking the inverse transform of Equation (3) results in Equation (4).

$$\int_{0}^{t_{k}} y(t)dt + a_{n-1} \int_{0}^{t_{k}} \int_{0}^{t_{k}} y(t)dt^{2} +$$

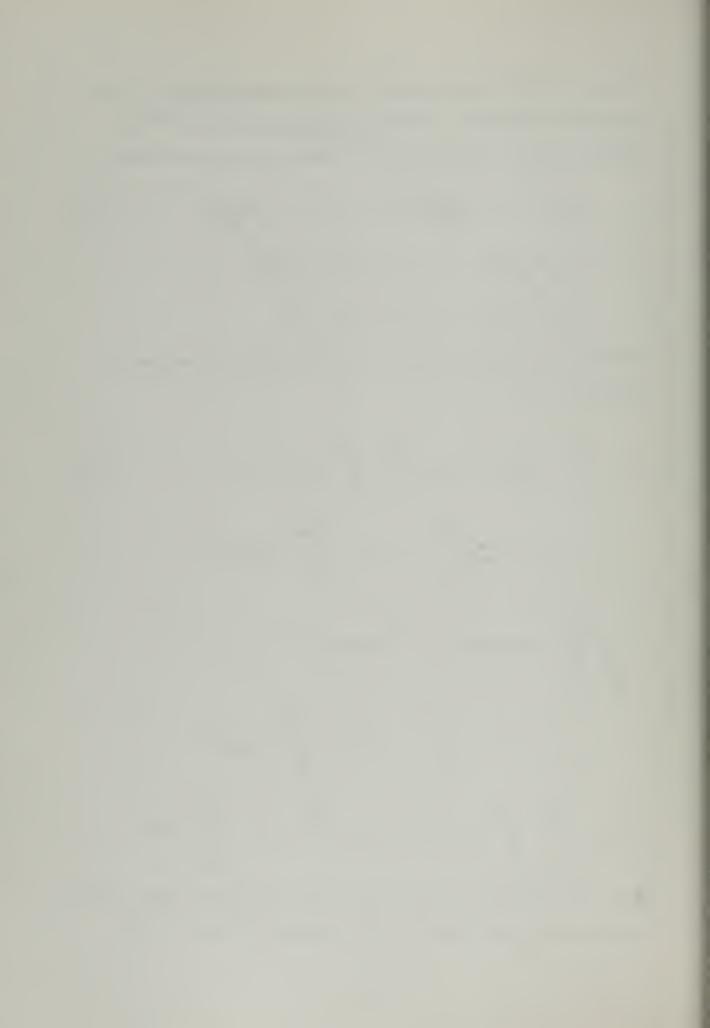
$$\cdots + a_{0} \int_{0}^{t_{k}} \cdots n+1 \cdots \int_{0}^{t_{k}} y(t)dt^{n+1}$$

$$= b_{m} \int_{0}^{t_{k}} \cdots n-m+1 \cdots \int_{0}^{t_{k}} u(t)dt^{n-m+1} + \cdots$$

$$\cdots + b_{0} \int_{0}^{t_{k}} \cdots n+1 \cdots \int_{0}^{t_{k}} u(t)dt^{n+1}$$

$$+ g_{n-1} \int_{0}^{t_{k}} \int_{0}^{t_{k}} dt^{2} + \cdots + g_{0} \int_{0}^{t_{k}} \cdots n+1 \cdots \int_{0}^{t_{k}} dt^{n+1} .$$

Since the system is time invariant nothing has been lost by setting the lower limit on the integrals equal to zero.



Rearranging terms in Equation (4) and placing all terms which depend on u(t), y(t), or t in brackets results in Equation (5).

$$a_{0} \int_{0}^{t_{k}} \dots n+1 \dots \int_{0}^{t_{k}} y(t) dt^{n+1} + \dots + a_{n-1} \int_{0}^{t_{k}} \int_{0}^{t_{k}} y(t) dt^{2}$$

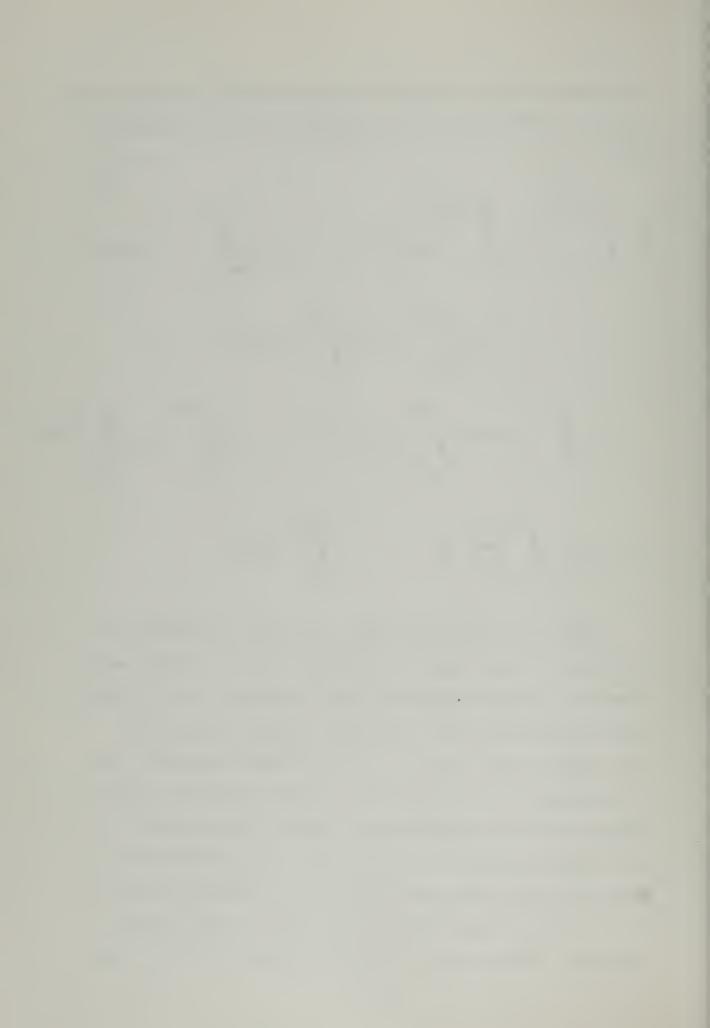
$$+ b_{0} \int_{0}^{t_{k}} \dots n+1 \dots \int_{0}^{t_{k}} u(t) dt^{n+1} + \dots$$

...
$$b_{m} \int_{0}^{t_{k}} ... -m+1... \int_{0}^{t_{k}} u(t) dt^{n-m+1} + g_{0} \int_{0}^{t_{k}} ... +1... \int_{0}^{t_{k}} dt^{n+1}$$

$$t_{n-1} \int_{0}^{t_{k}} \int_{0}^{t_{k}} dt^{2} = -\int_{0}^{t_{k}} y(t) dt$$

Since records of the input and output are assumed to be known a linear algebraic equation with the system parameters and initial condition terms as unknowns can be formulated by performing the indicated multiple integrations from zero to some time t_k . A set of 2n+m+1 equations can be obtained by letting t_k take on 2n+m+1 different values. Assuming that the equations are linearly independent it will now be possible to solve for the n+m+1 differential equation coefficients and the n initial condition terms.

So far nothing has been said of how n and m are determined. Theoretically it should be possible to use any



n' and m' greater than the actual order of the system under study. If the order of the system is n with m input coefficients then one would expect the following:

$$a_i = 0$$
 for $(0 \le i \le n'-n-1)$
 $b_i = 0$ for $(0 \le i \le m'-m-1)$
with n'>n and m'>m.

The model should essentially reduce itself to the right order by setting nonessential terms equal to zero.

Unfortunately the situation is not quite this simple. Due to the finite accuracy of all experimental data the linear equations will not have an exact solution. For certain types of inputs the linear equations will not even be linearly independent. These problems can be overcome to some extent by formulating more than 2n+m+1 equations which are required. The overdetermined set can then be solved using the method of least squares. If the limited accuracy of the experimental data can be attributed to round off errors then integrating the data over a time interval much greater than the sampling interval should remove much of the uncertainty in the linear equation coefficients. This is because the roundoff process can be modeled as a zero mean white noise process. The integral of the noise will approach zero as the period of integration increases.

Even the measures mentioned above will not solve the problem completely however. Due to the finite precision used to represent numbers in a computer and the iterative



nature of the numerical methods used to solve overdetermined sets of linear equations it is impossible to obtain zero as a solution for any parameter. If the parameter should be zero the numerical method will return a very small but nonzero value. Although this will result in an incorrect estimate of the system order the error will not be serious in most cases. This is because the small coefficients of the terms which should nonexistent will make their effect negligible. Examples in Chapter III will demonstrate this point.

Although the development in this section has assumed a differential equation model of the system the same results could have been obtained if a transfer function or state variable model had been assumed. A simple rearrangement of terms in Equation (2) results in Equation (6).

$$\frac{Y(s)}{U(s)} = \frac{b_m s^m + \cdots + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}.$$
 (6)

By defining a few new terms, Equation (7) results.

$$\frac{Y(s)}{U(s)} = \frac{K(s^{m} + c_{m-1}s^{m-1} + \cdots + c_{1}s + c_{0})}{s^{n} + a_{n-1}s^{n-1} + \cdots + a_{1}s + a_{0}}$$
(7)

where $K = b_{m}$

$$c_i = b_i / b_m$$
 for $0 \le i \le m$.

A set of state equations may be formulated in a similar fashion. One convenient state variable representation is given in Equation (8). It is based on the phase variable form of system representation.



$$\begin{bmatrix} \dot{x}_{1} \\ \dot{x}_{2} \\ \vdots \\ \vdots \\ \dot{x}_{n} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & & & \\ 0 & 0 & \ddots & & \\ \vdots & \ddots & \ddots & \ddots & \\ 0 & 0 & 1 & & \\ \vdots & \ddots & \ddots & \ddots & \\ 0 & 0 & 1 & & \\ \vdots & \ddots & \ddots & \ddots & \\ 0 & 0 & 0 & 1 & \\ \vdots & \ddots & \ddots & \\ x_{n} \end{bmatrix} + \begin{bmatrix} 0 & & & & \\ 0 & & & \\ \vdots & & & \\ \ddots & & & \\ x_{n} \end{bmatrix}$$

$$y = \begin{bmatrix} c_{0} & c_{1} & \cdots & c_{m} & 0 & \cdots & 0 \end{bmatrix} \times \begin{bmatrix} x_{1} \\ x_{2} \\ \vdots \\ x_{n} \end{bmatrix}$$

Note the simple correspondence between the terms in the transfer function and the terms in the state equations.

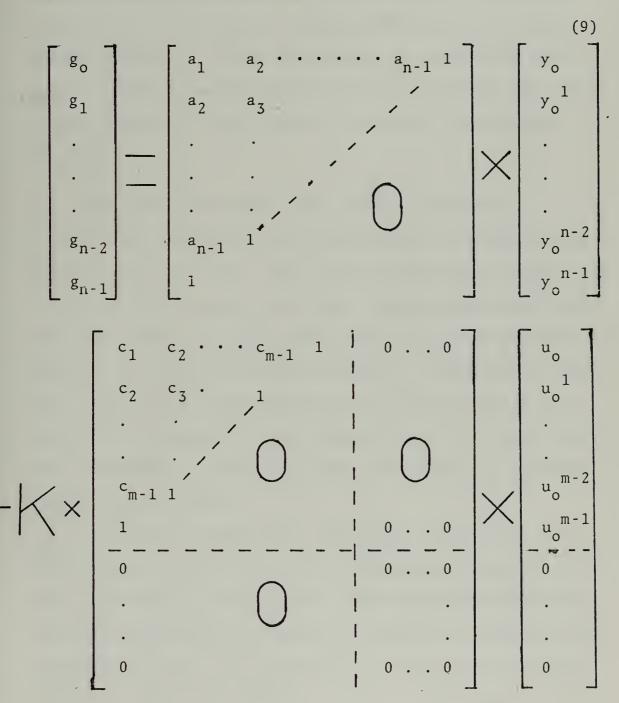
B. IDENTIFYING INITIAL CONDITIONS

In many identification problems it is desirable to compare the system model with the actual system by exciting the system model with the same input data that was used in the identification. By comparing the output of the model with the output of the system a rough idea of the accuracy of the model can be obtained. This will not be possible however unless the initial conditions at the beginning of the input-output record are all known.

From Equation (5) it can be seen that when the linear algebraic equations are solved for the unknown model parameters



the g_i initial condition terms are also found. By taking the Laplace Transform of Equation (1) and specifically writing in the contributions of the individual initial conditions a relationship between the g_i terms and the initial conditions can be found.





Unfortunately Equation (9) requires the knowledge of m-1 derivatives of the input function u. It may be necessary to calculate m-1 derivatives of the input using numerical techniques. This could cause the model and system output to differ slightly at the beginning of the output record but as the natural response dies out the records should converge. It may be possible to avoid this difficulty in many cases by choosing the beginning of the inputoutput record at a point where the input is relatively constant.

C. SIMPLIFICATIONS WITH ZERO INITIAL CONDITIONS

In many problems it will be possible to exercise complete control over the input to the system under study. If it is possible to obtain an input-output record beginning when the system is in the zero state it will be possible to simplify the identification procedure. Since the g_i terms will all be zero if the system is in the zero state they need not be included in the formulation of the linear algebraic equations. This will reduce the number of unknowns from 2n+m+1 to n+m+1.

It should be noted that additional information can often be incorporated into the identification procedure in order to simplify the problem. For example if the steady state gain constant were known the number of unknowns could be reduced by one. It is usually a simple matter to tell whether a system has poles or zeroes at the origin. This



piece of information can easily be used to simplify the identification procedure still further. As a general rule, the fewer the unknowns the more accurate the identification.



III. IMPLEMENTATION

A. NUMERICAL METHODS

The identification technique presented in Chapter II can be broken into two basic steps. The first step consists of performing the multiple integrations of the input and output and forming the overdetermined set of linear algebraic equations. The second step consists of solving the linear equations for the unknown model parameters. It is a distinct advantage of the identification technique under study that each of these steps can be carried out by subprograms that are readily available in virtually all modern computer centers.

Step one can be handled by any numerical integration subroutine. Although there are quite a few highly sophisticated numerical integration procedures available, trapezoidal integration will give better results in most applications. There are several reasons why this is true. First of all, most of the more complex integration techniques perform poorly when the function being integrated is discontinuous. Since control system inputs are often discontinuous and since such discontinuities are quite desirable from an identification standpoint, complex integration techniques are usually undesirable. Even when the functions to be integrated are continuous the slight improvement in accuracy offered by more advanced methods is not enough to justify the tremendous increase in computational load associated with their use.



Step two, the solution of the set of overdetermined linear equations, is a classical problem in several fields of mathematics and engineering. Unfortunately most of the classical techniques for solving such problems are not practical. They tend to magnify the errors introduced by the finite precision of the computer to the point where the solution is meaningless. Fortunately several modern methods are available that display more acceptable behavior. The method used in this paper was developed by Golub [31] The basic approach is to triangularize the coefficient matrix by performing a Choleski decomposition. decomposition is accomplished by applying repeated Householder transformations [32]. Once the coefficient matrix has been triangularized the unknowns can be obtained by back substitution. The method is quite stable numerically and is capable of handling illconditioned coefficient matrices.

B. CHARACTERISTICS OF GOOD INPUT-OUTPUT RECORDS

The accuracy with which a system can be identified is strongly dependent on the input-output record used in the identification. Since parameters are identified on the basis of their effect on the output it will be impossible to identify a parameter unless its effect is measureable. If the input to a system has a frequency spectrum that is more or less uniform over the frequency range of interest the identification will probably be very good. If the



frequency spectrum of the input is confined to a narrow band of frequencies the identification will probably be very bad. It is well known that signals with sharp discontinuities have a broader bandwidth than slowly varying continuous signals. For this reason input-output records displaying discontinuities and rapid time variations should be chosen.

If step or ramp inputs are used in the identification the value of the initial conditions will have to be known and incorporated into the set of linear equations. Since the initial conditions will usually be zero when these types of inputs are used this should not cause any difficulties. The reason for this difficulty lies in the nature of the initial condition coefficient terms. The integral coefficients of these terms are steps, ramps, and higher order polynomials in time. If the input is a step or a ramp the coefficients of several model parameters will also be steps, ramps, and higher order polynomials in time. There will therefore be a direct relationship between model parameter coefficients and initial condition term coefficients. This will result in the linear equations having an infinite number of solutions due to the linear dependence between all the individual equations in the set. Step and ramp inputs must therefore be avoided when the system initial conditions are unknown.

Since analog system data will have to be converted to digital form a suitable sampling interval will have to be



chosen. Experimental results have shown that a sampling rate ten to one hundred times shorter than the shortest system time constant works quite well. Lower sampling rates may introduce inaccuracies in the location of high frequency poles and zeroes.

C. PROGRAM DESCRIPTION

In order to evaluate experimentally the characteristics of the identification procedure under study a set of digital computer programs was developed. The main identification program is a direct implementation of the procedure developed in Chapter II. Subroutine SYSTEM is a simulation program that was written to generate input-output data for the main program to process.

In the beginning of the identification program several important parameters are defined. NP and NZ are rough estimates of the number of poles and the number of zeroes in the system to be identified. KPTMAX is the number of sample points available in the input-output record. Each sample point will consist of the time (T), the input amplitude (R), and the output amplitude (C). IPTS is the number of sample points that will separate successive linear equations. In other words, every time the total number of points read in is a multiple of IPTS a linear equation will be generated. The total number of linear equations that will be generated is equal to KPTMAX/IPTS. When all of the linear equations have been formed subroutine DLLSQ is called. This



subroutine is an implementation of the Golub algorithm for solving overdetermined sets of linear equations.

Subroutine DLLSQ returns the values of the system model parameters and initial condition parameters of Equation (2). In order to find the poles and zeroes of the system the output of DLLSQ is fed into RTPLSB. RTPLSB is a polynomial root finder which uses a combination of the Newton-Raphson and Bairstow methods to find the poles and zeroes of the system.

The remainder of the main identification program is devoted to output. The results of the identification are given in both transfer function and state variable form.

The state variable form is referenced to the format used in Equation (8).

Subroutine SYSTEM reads in the transfer function of a system and computes the system output based on a set of arbitrary initial conditions and an arbitrary input waveform. Each time subroutine SYSTEM is called by the identification program it returns three numbers, the time T, the input waveform amplitude R, and the output waveform amplitude C. Subroutine SYSTEM prints out the transfer function and state variable representation of the system it is simulating so that the accuracy of the identification can be determined.

Input-output recrods obtained from physical systems are rarely accurate to more than three or four significant digits. The data generated by subroutine SYSTEM is therefore



rounded off by subroutine ROUND before being passed to the identification program. The number of significant digits in the data returned to the identification program may be varied by changing the value of the parameter NA in the simulation subroutine.

D. EXAMPLES

The following examples demonstrate how the accuracy of an identification depends on the accuracy of the inputoutput record, the sampling period, and the input waveform.
They also show how the order of the system can be determined from a trial identification run using an estimated
order greater than the actual order of the system.

Example one illustrates the relationship between the accuracy of the input-output record data and the resulting identification. In order to minimize the effect of other factors all initial conditions were set equal to zero, a step input was used, and n' and m' were set equal to n and m. Example one demonstrates the fact that there is a direct (almost linear) relationship between the accuracy of input-output data and the accuracy of the identification. Note that even when the input-output record contained only two significant digits the identification of the system poles and zeroes was within about three percent of their exact values.

Example two illustrates the relationship between the sampling period used with the input-output records and the



the accuracy of the identification, input records containing discontinuities or rapid time variations should be chosen.

Examples four through ten demonstrate what happens when n' and m' are greater than n and m. In each example an identification is performed using an n' and m' greater than n and m. Using the information obtained from this identification a new value of n' and m' is determined. These new estimates are then used to perform a second identification.

In examples four and five the error in the estimate of m' was made larger than the error in the estimate of n'. As a result of the relative values of these two errors all the excess poles cancelled with excess zeroes but since there were more excess zeroes than excess poles some excess zeroes remained. Note however that for frequencies lower than the sampling rate the excess zeroes have little or no effect on the behavior of the identified system. Experiments have shown that excess zeroes that do not cancel with excess poles are always of a frequency comparable to or higher than the sampling rate. Using this principle and estimating new values for n' and m' results in an identification which has the correct number of poles and zeroes and is more accurate than the first identification.

In examples six, seven, and eight the error in the estimate of n' was equal to the error in the estimate of m'. As a result, all excess poles and zeroes cancelled



with each other leaving a system of the correct order.

Note that by repeating the identification with the correct value of n' and m' it was possible to improve the accuracy of the identification.

In examples nine and ten the error in the estimate of n' was made greater than the error in the estimate of m'. As a result, all excess zeroes cancelled with excess poles but some excess poles remained. Note that for frequencies lower than the sampling rate the excess poles have negligible effect. Experiments have shown that excess poles that do not cancel with excess zeroes are almost always of a frequency comparable to or greater than the sampling rate. Using this principle and estimating new values for n' and m' resulted in a correct identification of the simulated systems.

Experiments have shown that identifications involving uncancelled excess zeroes are generally more accurate than identifications involving uncancelled excess poles. For this reason it is best to set m' close to n' when identifying an unknown system.

Using the experimental findings described above a simple procedure for determining n and m can be formulated. First, guess an n' which is greater than the order of the system to be identified. This should not be too difficult in most engineering identification problems. Let m' be equal to n' or m'-1. This will guarantee complete cancellation of all excess poles and zeroes or partial cancellation



of excess poles and zeroes with excess zeroes remaining.

After making a trial identification with the values of n' and m' mentioned above, estimate new values for n' and m' based on the reasoning in the examples. The new values of n' and m' should now be correct. By performing the identification with these values of n' and m' a good identification should result.



EXAMPLE 1.

SYSTEM TO BE IDENTIFIED

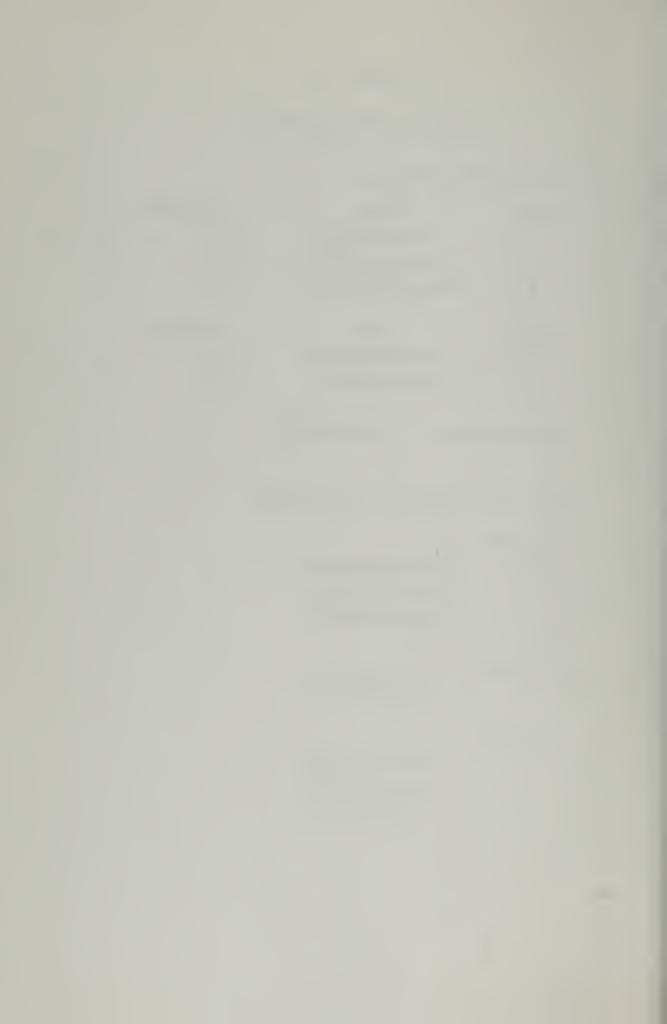
SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINA	ARY
1	-1.0000000000	0.0	J
2	-10.0000000000	0.0	J
3	-100.0000000000	0.0	J
ZEROES	REAL	IMAGIN	ARY
1	-3.0000000000	0.0	J
2	-30.0000000000	0.0	J

GAIN CONSTANT = 10.000000000

SYSTEM STATE VARIABLES (PHASE FORM)

Α	VECTOR	
	1	1000.00000000000
	2	1110.0000000000
	3	111.0000000000
В	VECTOR	
	3	10.000000000
С	VECTOR	
	1	90.0000000000
	2	33.000000000
	3	1.000000000



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINA	RY
1	-100.0001690225	0.0	J
2	-0.9999970711	0.0	J
3	-9.9999884241	0.0	J
ZEROES	REAL	IMAGINA	RY
1	-2.9999937654	0.0	J
2	-30.0000138488	0.0	J

GAIN CONSTANT = 10.0000038147

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR	
1	999.9976037596
2	1110.0003679024
3	111.0001545177
B VECTOR	
3	10.0000038147
C VECTOR	
1	89.9998545091
2	33.0000076143
3	1.0000000000

PROGRAM PARAMETERS

RECORD LEGNTH = 0.60000 SEC SAMPLING PERIOD = 0.20000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 8 DECIMAL PLACES



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY	
1	-100.0000291125	0.0	J
2	-1.0000011522	0.0	J
3	-10.0000016550	0.0	J
ZEROES	REAL	IMAGINARY	
1	-3.0000028353	0.0	J
2	-30.0000250480	0.0	J

GAIN CONSTANT = 9.9999933243

SYSTEM STATE VARIABLES (PHASE FORM)

Α	٧	E	C	T	0	R	

1	1000.0016088045
2	1110.0006141285
3	111-0000319197

B VECTOR

3 9.9999933243

C VECTOR

1	90.0001602028
2	33.0000278833
3	1.0000000000

PROGRAM PARAMETERS

RECORD LEGNTH = 0.60000 SEC SAMPLING PERIOD = 0.20000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 7 DECIMAL PLACES



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGIN	ARY
1	-100.0000683416	0.0	J
2	-1.0000118079	0.0	J
3	-10.0000329083	0.0	J
ZEROES	REAL	IMAGIN	IARY
1	-3.0000272949	0.0	J
2	-30.0000668753	0.0	J

GAIN CONSTANT = 9.9999847412

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR

1 1000.0157822215 2 1110.0053743721 3 111.0001130579

B VECTOR

3 9.9999847412

C VECTOR

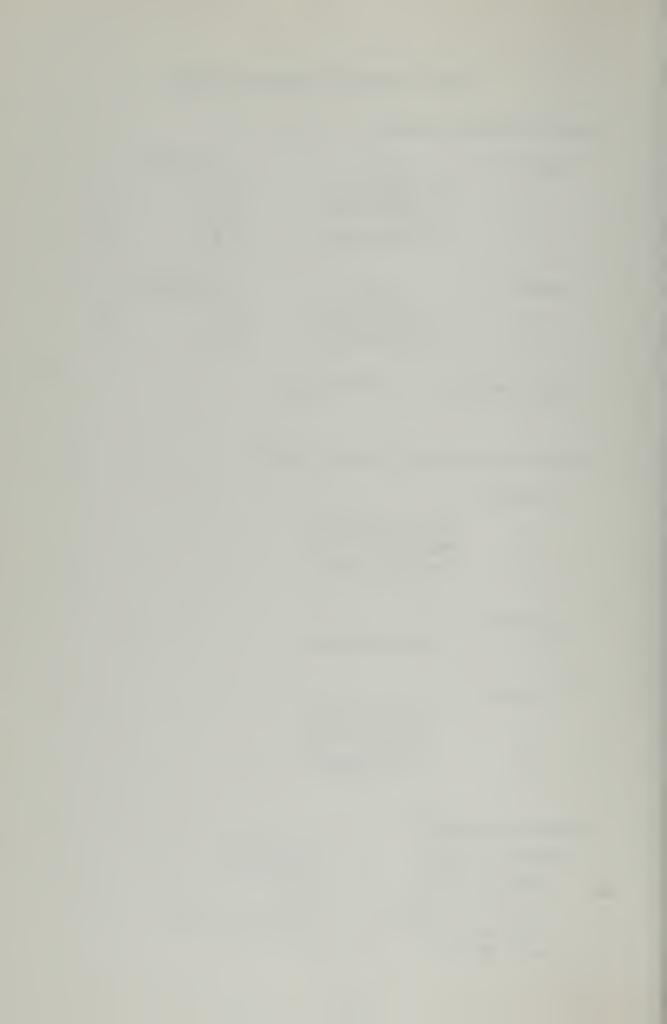
1 90.0010194754 2 33.0000941702 3 1.0000000000

PROGRAM PARAMETERS

RECORD LEGNTH = 0.60000 SEC SAMPLING PERIOD = 0.20000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 6 DECIMAL PLACES



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGIN	IARY
1	-99.9968491528	0.0	J
2	-1.0000017165	0.0	J
3	-9.9998390264	0.0	J
ZEROES	REAL	IMAGIN	IARY
1	-2.9999754422	0.0	J
2	-29.9992863605	0.0	J

GAIN CONSTANT = 9.9998254776

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTUR	
1	999.9541111369
2	1109.9492716708
2	110 0066808058

B VECTOR

3 9.9998254776

C VECTOR

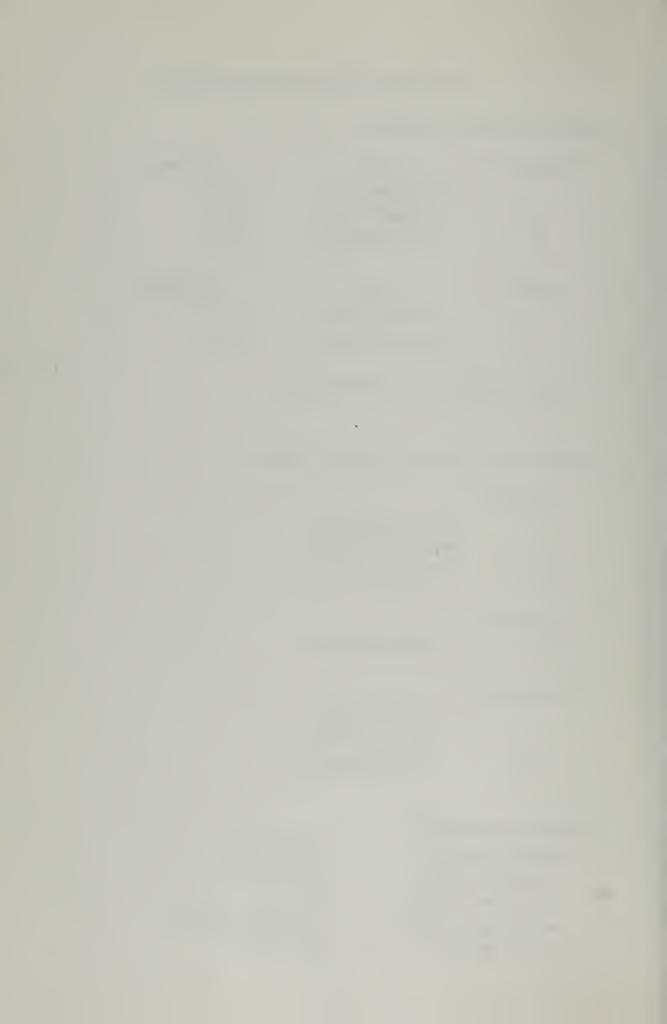
1 89.9971223661 2 32.9992618027 3 1.0000000000

PROGRAM PARAMETERS

RECORD LEGNTH = 0.60000 SEC SAMPLING PERIOD = 0.20000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 5 DECIMAL PLACES



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGIN	ARY
1	-99.9927835443	0.0	J
2	-0.9991304951	0.0	J
3	-9.9980805636	0.0	J
ZEROES	REAL	IMAGIN	ARY
1	-2.9981092090	0.0	J
2	-29.9981888298	0.0	J

GAIN CONSTANT = 9.9994869232

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR

1 998.8666303721 2 1109.6311321656 3 110.9899946030

B VECTOR

3 9.9994869232

C VECTOR

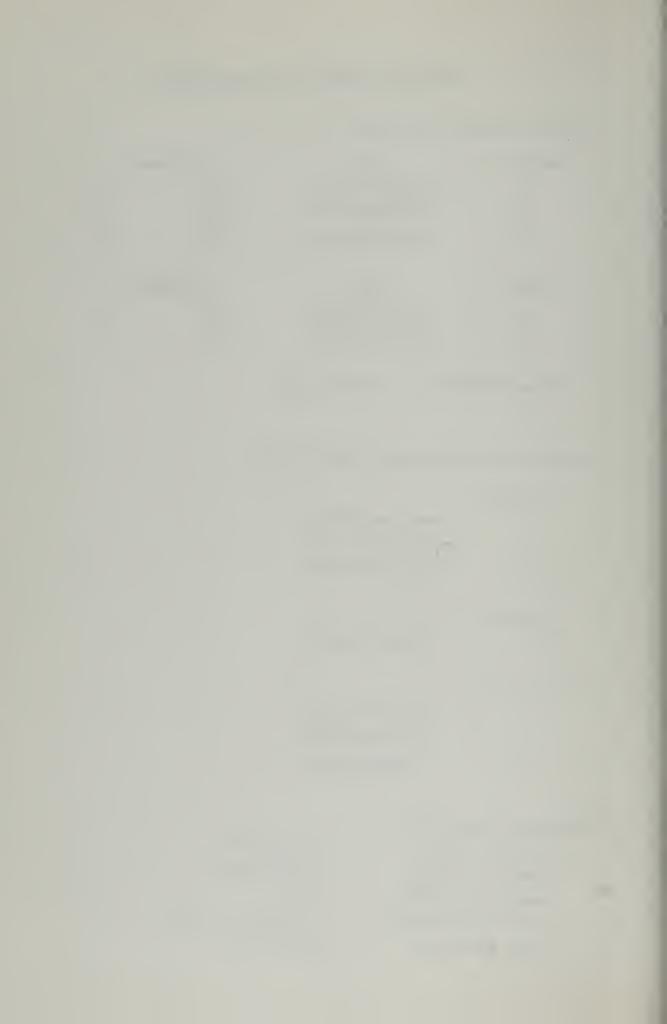
1 89.9378461845 2 32.9962980388 3 1.0000000000

PROGRAM PARAMETERS

RECORD LEGNTH = 0.60000 SEC SAMPLING PERIOD = 0.20000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 4 DECIMAL PLACES



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGIN	ARY
1	-99.9937910594	0.0	J
2	-0. 9996622783	0.0	J
3	-10.0015833055	0.0	J
ZEROES	REAL	IMAGIN	ARY
1	-2.9996507024	0.0	J
2	-30.0061026888	0.0	J

GAIN CONSTANT = 9.9986057281

SYSTEM STATE VARIABLES (PHASE FORM)

Α	VECTOR	
	1	999.7584771470
	2	1110.0544578539
	3	110.9950366432
В	VECTOR	
	3	9.9986057281
С	VECTOR	
	1	90.0078270058
	2	33.0057533912
	3	1.0000000000

PROGRAM PARAMETERS

RECORD LEGNTH = 0.60000 SEC SAMPLING PERIOD = 0.20000 MSC EQUATIONS FORMED = 120

PRECISION OF DATA = 3 DECIMAL PLACES



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGIN	ARY
1	-98.8004578528	0.0	J
2	-0.9763637189	0.0	J
3	-9.8862584052	0.0	J
ZEROES	REAL	IMAGIN	ARY
1	-2.9406610975	0.0	J
2	-29.6497245014	0.0	J

GAIN CONSTANT = 9.9451894760

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR 953.6797208413 2 1082.8846233636 3 109.6630799769 B VECTOR 3 9.9451894760 C VECTOR 87.1897913926 1 2 32.5903855989 3 1.0000000000

PROGRAM PARAMETERS

RECORD LEGNTH = 0.60000 SEC SAMPLING PERIOD = 0.20000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 2 DECIMAL PLACES



EXAMPLE 2.

SYSTEM TO BE IDENTIFIED

SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINA	LRY
1	-1.0000000000	0.0	J
2	-10.0000000000	0.0	J
3	-100.0000000000	0.0	J
ZEROES	REAL	IMAGINA	ARY
1	-3.0000000000	0.0	J
2	-30.0000000000	0.0	J

GAIN CONSTANT = 10.0000000000

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR	
1	1000.0000000000
2	1110.0000000000
3	111.0000000000
B VECTOR	
3	10.0000000000
C VECTOR	
1	90.0000000000
2	33.0000000000
3	1.0000000000



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINA	ARY
1	-101.0610717819	0.0	J
2	-1.0000331312	0.0	J
3	-10.0017011716	0.0	J
ZEROES	REAL	IMAGINA	ARY
1	-3.0002183624	0.0	J
2	-30.0553440030	0.0	J

GAIN CONSTANT = 10.0887174606

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR	
1	1010.8161284948
2	1121.8490926385
3	112.0628060847
B VECTOR	
3	10.0887174606
C VECTOR	
1	90.1725949663
2	33.0555623654
3	1.0000000000

PROGRAM PARAMETERS

RECORD LEGNTH = 0.60000 SEC SAMPLING PERIOD = 0.10000 MSC EQUATIONS FORMED = 120

PRECISION OF DATA = 3 DECIMAL PLACES



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINA	RY
1	-105.3244028971	0.0	J
2	-1.0000069733	0.0	J
3	-10.0258525504	0.0	J
ZEROES	REAL	IMAGINA	RY .
1	+3.0005906385	0.0	J
2	-30.4710561855	0.0	J

GAIN CONSTANT = 10.3944911957

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR

1 1055.9742969792 2 1171.3179932211 3 116.3502624208

B VECTOR

3 10.3944911957

C VECTOR

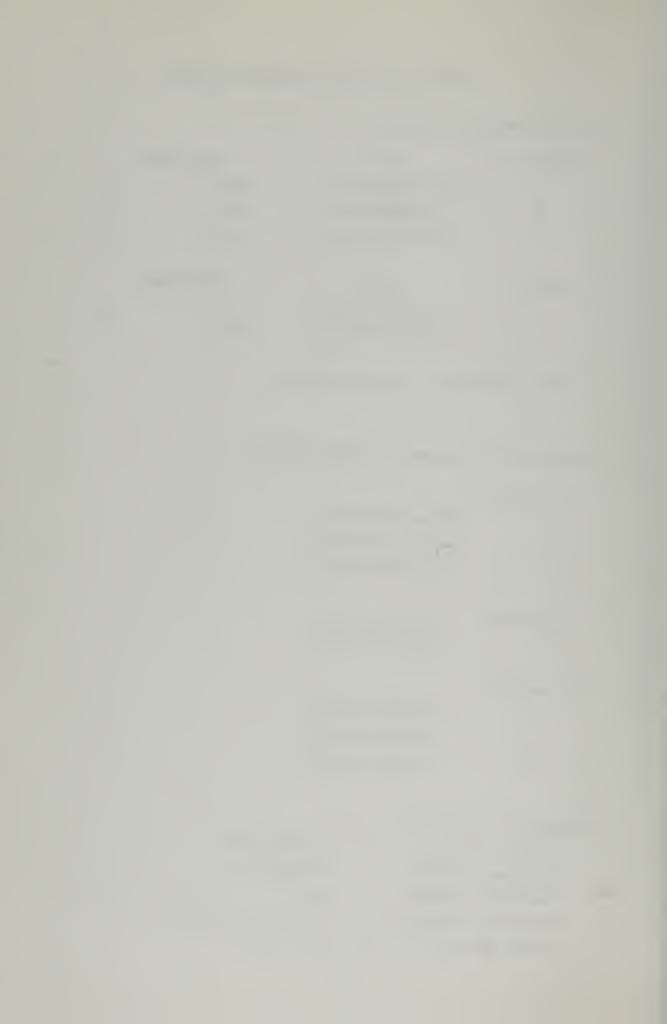
1 91.4311659362 2 33.4716468241 3 1.00000000000

PROGRAM PARAMETERS

RECORD LEGNTH = 1.20000 SEC SAMPLING PERIOD = 0.20000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 3 DECIMAL PLACES



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGIN	ARY
1	-107.6394221979	0.0	J
2	-1.0000162038	0.0	J
3	-10.0077785631	0.0	J
ZEROES	REAL	IMAGIN	ARY
1	-3.0001692900	0.0	J
2	-30.3047449110	0.0	J

GAIN CONSTANT = 10.6635513306

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR 1 1077.2489572831 2 1194.8806091150 3 118.6472169649 B VECTOR 3 10.6635513306 C VECTOR 1 90.9193650217

PROGRAM PARAMETERS

2

RECORD LEGNTH = 3.00000 SEC SAMPLING PERIOD = 0.50000 MSC EQUATIONS FORMED = 120

PRECISION OF DATA = 3 DECIMAL PLACES

33.3049142009

1.0000000000



EXAMPLE 3.

SYSTEM TO BE IDENTIFIED

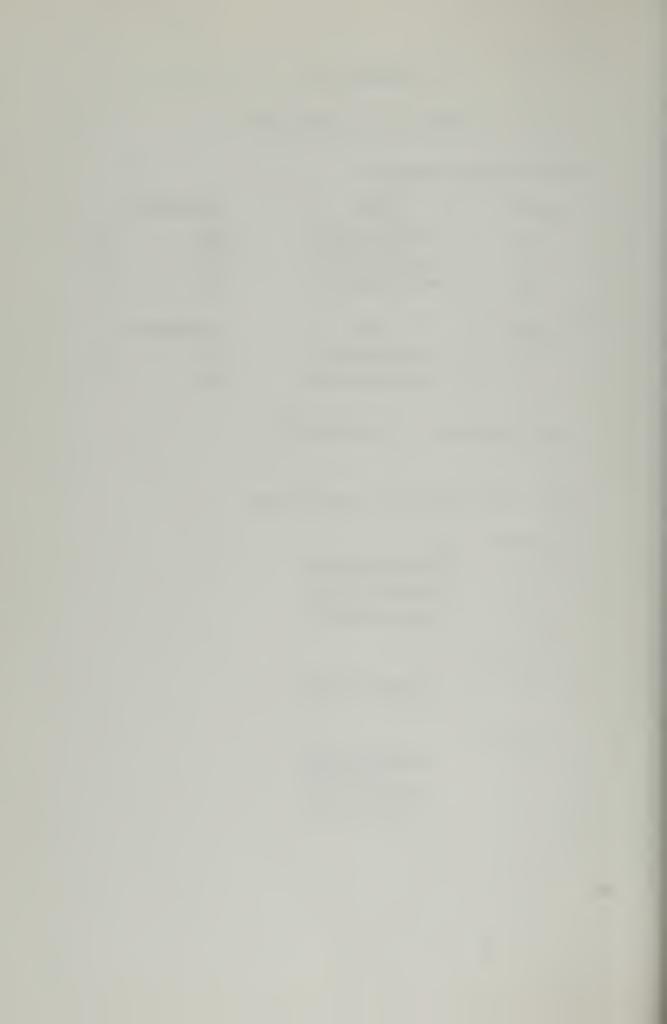
SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINAR	Y
1	-1.0000000000	0.0	J
2	-10.0000000000	0.0	J
3	-100.0000000000	0.0	J
ZEROES	REAL	IMAGINAR	Y
1	-3.0000000000	0.0	J
2	-30.000000000	0.0	J

GAIN CONSTANT = 10.0000000000

SYSTEM STATE VARIABLES (PHASE FORM)

Α	VECTOR	
	1	1000.0000000000
	2	1110.00000000000
	3	111.0000000000
В	VECTOR	
	3	10.0000000000
С	VECTOR	
	1	90.0000000000
	2	33.0000000000
	3	1.0000000000



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINA	RY
1	-99.6074714180	0.0	J
2	-1.0627466213	0.0	J
3	-10.0526991957	0.0	J
ZERGES	REAL	IMAGINA	RY
1	-3.1109954642	0.0	J
2	-29.8941214051	0.0	J

GAIN CONSTANT = 9.9883327484

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR 1 1064.1536423652 2 1117.8649236164 3 110.7229172350

B VECTOR 3 9.9883327484

C VECTOR	
1	93.0004760964
2	33.0051168692
3	1.0000000000

PROGRAM PARAMETERS

RECORD LEGNTH = 0.60000 SEC SAMPLING PERIOD = 0.10000 MSC EQUATIONS FORMED = 120

PRECISION OF DATA = 3 DECIMAL PLACES

INPUT FUNCTION = GAUSSIAN- SIGMA=0.010 SEC



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINA	ARY
1	-98.9033657608	0.0	J
2	-0.9389787239	0.0	J
3 .	-9.8317762550	0.0	J
Z EROES	REAL	IMAGINA	ARY
1	-2.8677895731	0.0	J
2	-29.6513565567	0.0	J

GAIN CONSTANT = 9.9508285522

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR

1 913.0589327259 2 1074.4957479197 3 109.6741207397

B VECTOR

3 9.9508285522

C VECTOR

1 85.0338511612 2 32.5191461297 3 1.0000000000

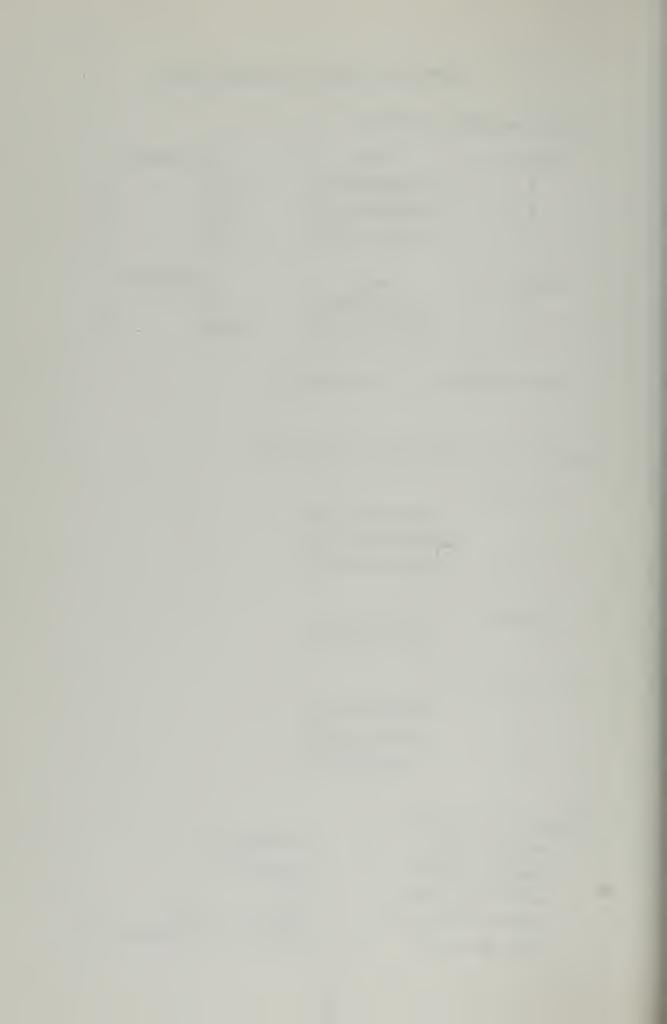
PROGRAM PARAMETERS

RECORD LEGNTH = 0.60000 SEC SAMPLING PERIOD = 0.10000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 3 DECIMAL PLACES

INPUT FUNCTION = GAUSSIAN- SIGMA=0.030 SEC



SYSTEM TRANSFER FUNCTION

REAL	IMAGINA	ARY
-99.1859265460	0.0	J
-0.7535004508	0.0	J
-9.5204582353	0.0	J
REAL	IMAGINA	ARY
-2.5186589038	0.0	J
-29.4122032416	0.0	J
	-99.1859265460 -0.7535004508 -9.5204582353 REAL -2.5186589038	-99.1859265460

GAIN CONSTANT = 9.9813451767

SYSTEM STATE VARIABLES (PHASE FORM)

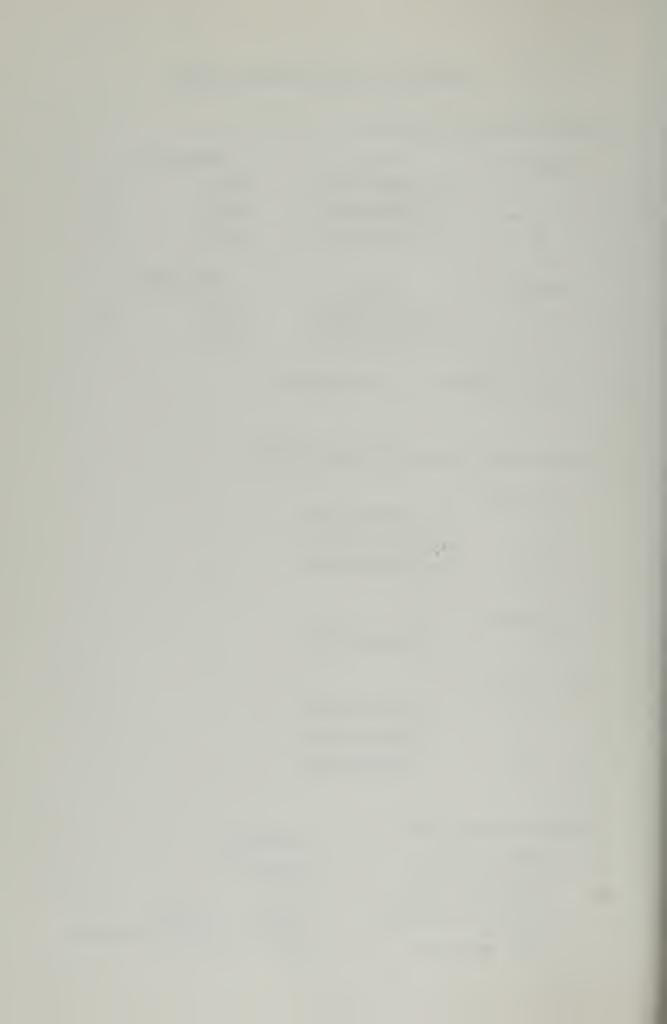
A	VECTOR	
	1	711.5270631979
	2	1026.2057811417
	3	109.4598852321
В	VECTOR	
	3	9.9813451767
С	VECTOR	
	1	74.0793075760
	2	31.9308621454
	3	1.0000000000

PROGRAM PARAMETERS

RECORD LEGNTH = 0.60000 SEC SAMPLING PERIOD = 0.10000 MSC EQUATIONS FORMED = 120

PRECISION OF DATA = 3 DECIMAL PLACES

INPUT FUNCTION = GAUSSIAN- SIGMA=0.050 SEC



EXAMPLE 4

SYSTEM TO BE IDENTIFIED

SYSTEM TRANSFER FUNCTION

POLES REAL IMAGINARY

1 -5.000000000 0.0 J

2 -20.000000000 0.0 J

ZEROES REAL I MAGINARY

GAIN CONSTANT = 300.000000000

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR

1 100.000000000 2 25.000000000

B VECTOR

2 300.000000000

C VECTOR

1 1.0000000000

INITIAL STATE VECTOR

1.0000000000

2 0.0



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY	
1	-20.1020603504	0.0	J
2	-0.4280895557	-6.7540893481	J
3	-0.4280895557	6.7540893481	J
4	1.0489471395	0.0	J
5	-4.9974142741	0.0	J
ZEROES	REAL	IMAGINARY	
1	21168.5357639489	0.0	J
2	-0.4681378779	-6.7682935708	J
3	-0.4681078779	6.7682935708	J
4	1.0470216652	0.0	J

GAIN CONSTANT = -0.0141926892

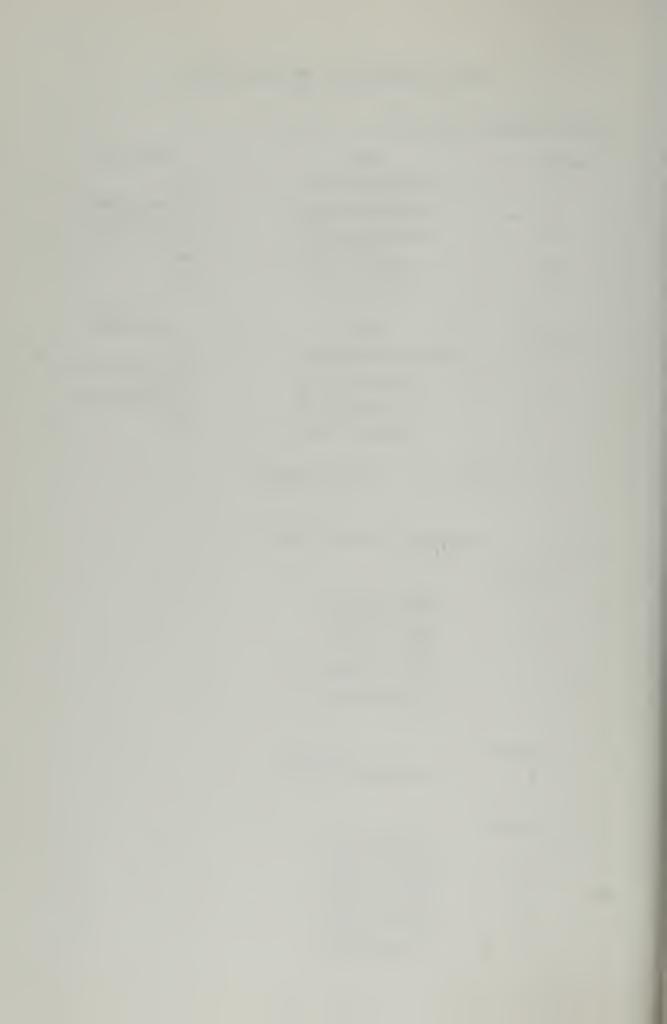
SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR	
1	-4826.300213
2	3305.020433
3	1059.6311.59
4 -	140.5228441
5	24.90670660
B VECTOR	
5	-0.1419268921E-01
C VECTOR	
1	1020181.159
2	-953662.8857
3	2390.647541

5

-21168.64657

1.000000000



INITIAL CONDITION (G) VECTOR

1 0. 9983029652

2 25.01148179

3 37.55868385

4 1118,728299

5 -1523.539669

PROGRAM PARAMETERS

RECORD LEGNTH = 3.00000 SEC

SAMPLING PERIOD = 0.50000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 3 DECIMAL PLACES

INPUT FUNCTION = PIECEWISE CONSTANT



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY
1	-4.9994757762	0.0 J
2	-20.0118644519	0.0 J

ZEROES REAL IMAGINARY

GAIN CONSTANT = 300.1457519531

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR

1 100.0488316 2 25.01134023

B VECTOR

2 300.1457520

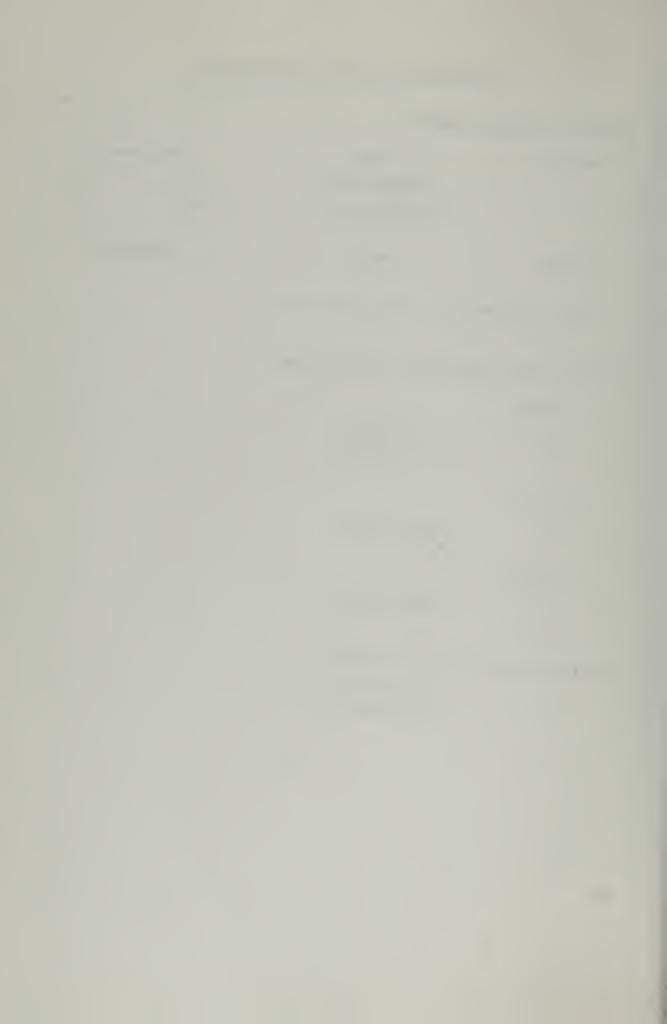
C VECTOR

1 1.000000000

INITIAL CONDITION (G) VECTOR

1 1.001249200

2 25.00380377



EXAMPLE 5

SYSTEM TO BE IDENTIFIED

SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY	
1	-2.0000000000	3.0000000000	J
2	-2.0000000000	-3.0000000000	J
3	-20.000000000	0.0	J
ZEROES	REAL	IMAGINARY	
1	-8.0000000000	0.0	J

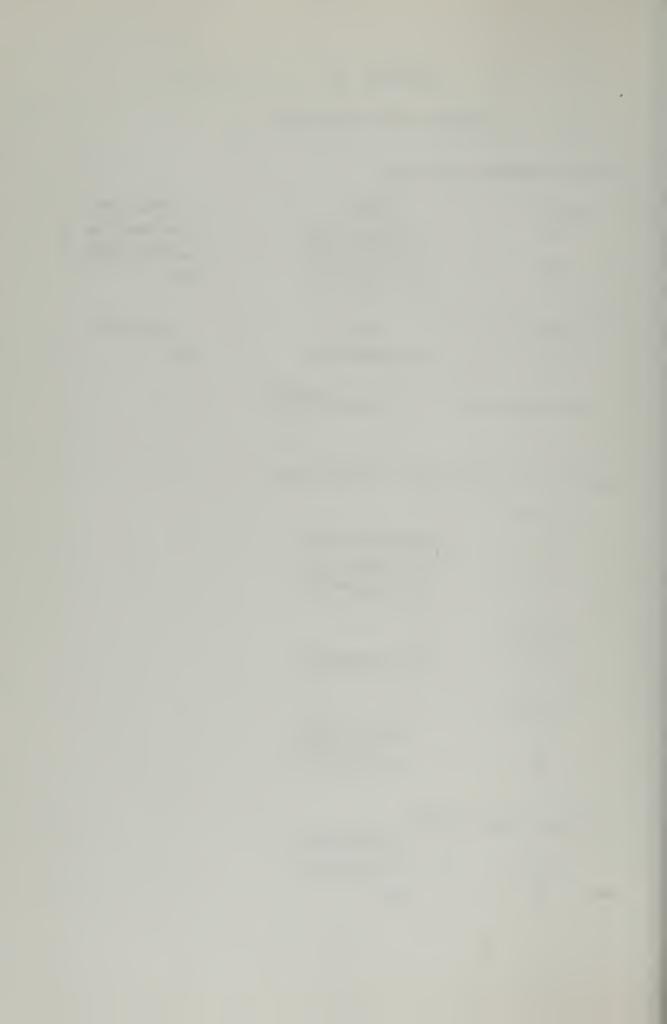
SYSTEM STATE VARIABLES (PHASE FORM)

GAIN CONSTANT = 160.0000000000

A VECTOR	
1	260.0000000000
2	93.0000000000
3	24.0000000000
B VECTOR	
3	160.0000000000
C VECTOR	
1	8.00000000000
2	1.0000000000

INITIAL STATE VECTOR

1	2.000000000
2	1.0000000000
3	0-0



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY	
1	-20.3839954707	0.0	J
2	1.0012938647	-8.8128722093	J
3	1.0012938647	8.8128722093	J
4	-1.9 998854 3 38	-3.0017839895	J
5	-1.9998854338	3.0017839895	J
ZEROES	REAL	IMAGINARY	
1	-7. 6038902797	0.0	J
2	447.1906330529	0.0	J
	1 00000004	-9.0382447685	.1
3	1.2922226696	7.0302111003	~

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR		
1	20863.16713	
2	6906.430945	
3	1994.127061	
4	124.3802348	
5	22.38117861	
B VECTOR		
5	-0.3623618484	•
	·	
C VECTOR		
1	-283455.3928	
2	-27855.73425	
3	-2180.940891	
4	-442.1711881	
5	1.0000000000	



INITIAL CONDITION (G) VECTOR

1 16.99753824

2 388.4269451

3 1684.166654

4 30704.56626

5 95381.34774

PROGRAM PARAMETERS

RECORD LEGNTH = 3.00000 SEC

SAMPLING PERIOD = 0.50000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 3 DECIMAL PLACES

INPUT FUNCTION = PIECEWISE CONSTANT



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY	
1	-20.1316142373	0.0	J
2	-1.9997989237	-3.0011201212	J
3	-1.9997989237	3.0011201212	J
Z EROES	REAL	IMAGINARY	
1	-8.0335677123	0.0	J

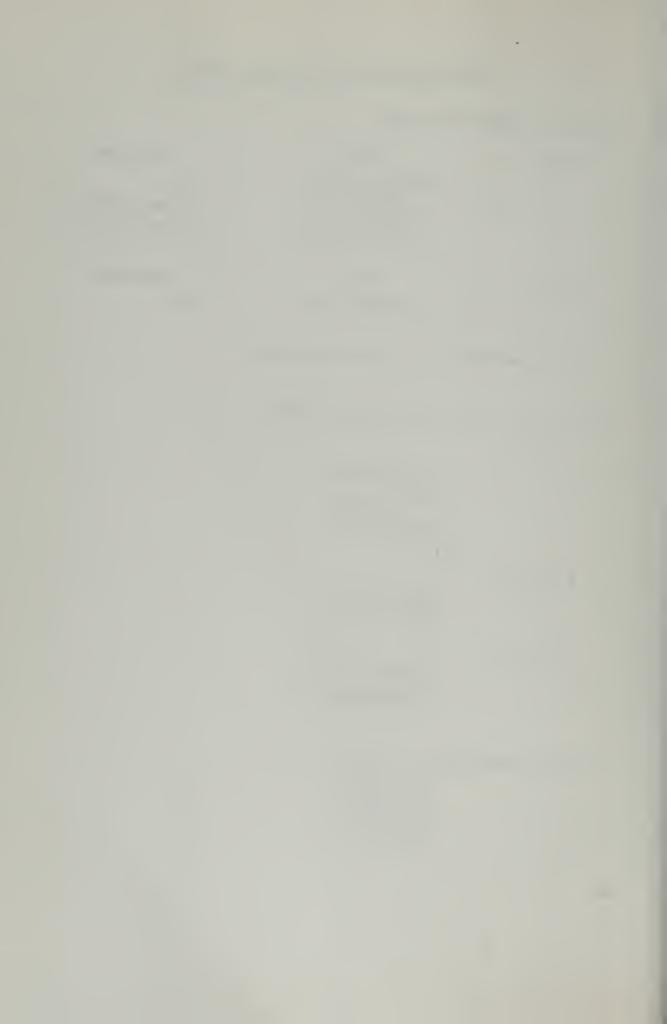
GAIN CONSTANT = 160.4525451660

SYSTEM STATE VARIABLES (PHASE FORM)

TOTEN SIA	TE VARIABLES (THAS	, , , , , , , , , , , , , , , , , , , ,
A VECTOR	₹	
1	261.8301183	
2	93.52427869)
3	24.13121208	3
B VECTOR	· ·	
3	160.4525452	
C VECTOR	₹	
1	8.033567712	2
2	1.000000000)
NITIAL CO	NDITION (G) VECTOR	. .
1	16.99920347	
2	418.2532176	5

3

1168.376804



SYSTEM TO BE IDENTIFIED

SYSTEM TRANSFER FUNCTION

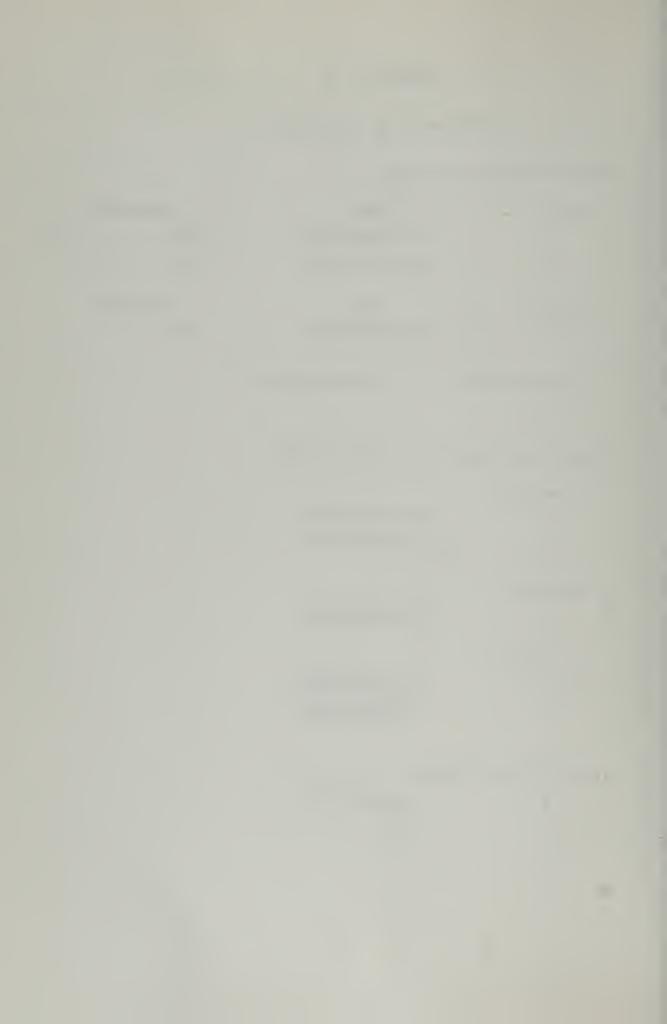
POLES	REAL	IMAGINARY	
1	-3.000000000	0.0	J
2	-45.0000000000	0.0	J
ZEROES	REAL	IMAGINARY	
1	-15.000000000	0.0	J.

SYSTEM STATE VARIABLES (PHASE FORM)

GAIN CONSTANT = 10.0000000000

A VECTOR	
1	135.00000000000
2	48.0000000000
B VECTOR	·
2	10.0000000000
C VECTOR	
1	15.0003000000
2	1.0000000000

1	1.0000000000	
2	0.0	



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY	
1	-44.9825756448	0.0	J
2	0.8248831729	-16.8802782130	J
3	0.8248831729	16.8802782130	J
4	-2.9985948695	0.0	J
5	-4.9097021934	0.0	J
ZEROES	REAL	IMAGINARY	
1	0.8362974237	-16.873983560	. J
2	0.8362974237	16.873983560	J
3	-15.0390073803	0.0	J
4	-4.9017160580	0.0	J

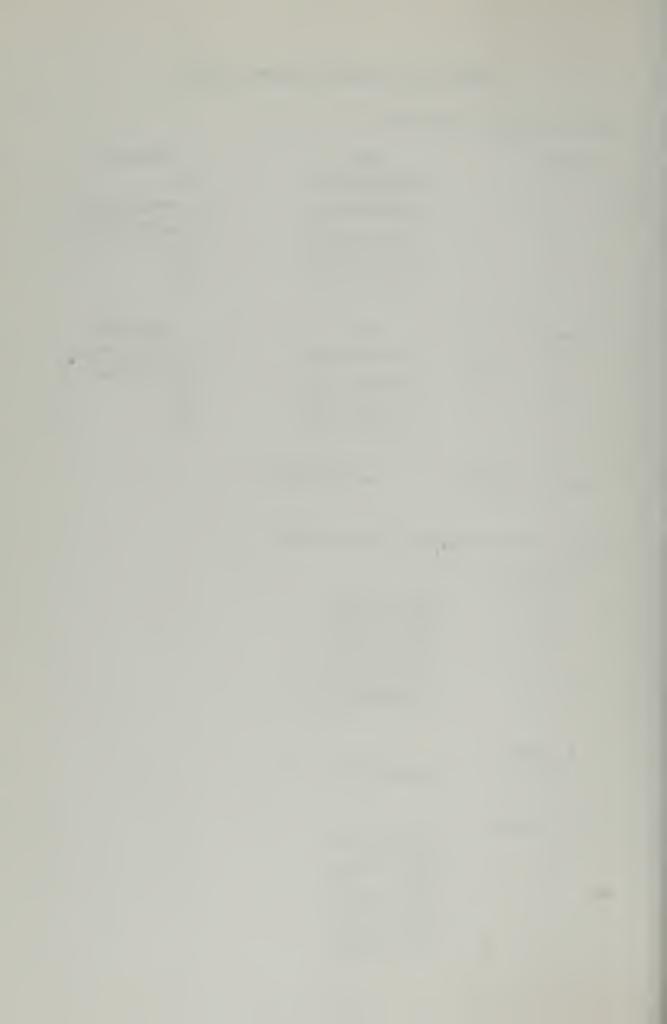
SYSTEM STATE VARIABLES (PHASE FORM)

GAIN CONSTANT = 9.9892787933

A VECTOR		
1	189152.5939	
2	104719.1699	
3	15157.98857	
4	568.8244217	
5	51.24110636	
B VECTOR		
5	9.989278793	•
C VECTOR		
1	21041.07999	
2	5568.396359	
3	325.7949073	
4	18.26812859	

5

1.000000000



INITIAL CONDITION (G) VECTOR

1 14.99958219

2 633.6563568

3 6068.388958

4 183314.0231

5 819995.9356

PROGRAM PARAMETERS

RECORD LEGNTH = 1.33333 SEC

SAMPLING PERIOD = 0.22222 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 4 DECIMAL PLACES



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY	
1	-2.9998001615	0.0	J
2	-44.9468936931	0.0	J
ZEROES	REAL	IMAGINARY	
1	-14.9948012787	0.0	J

GAIN CONSTANT = 9.9912204742

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR

1	134.8316990
2	47.94669385

B VECTOR

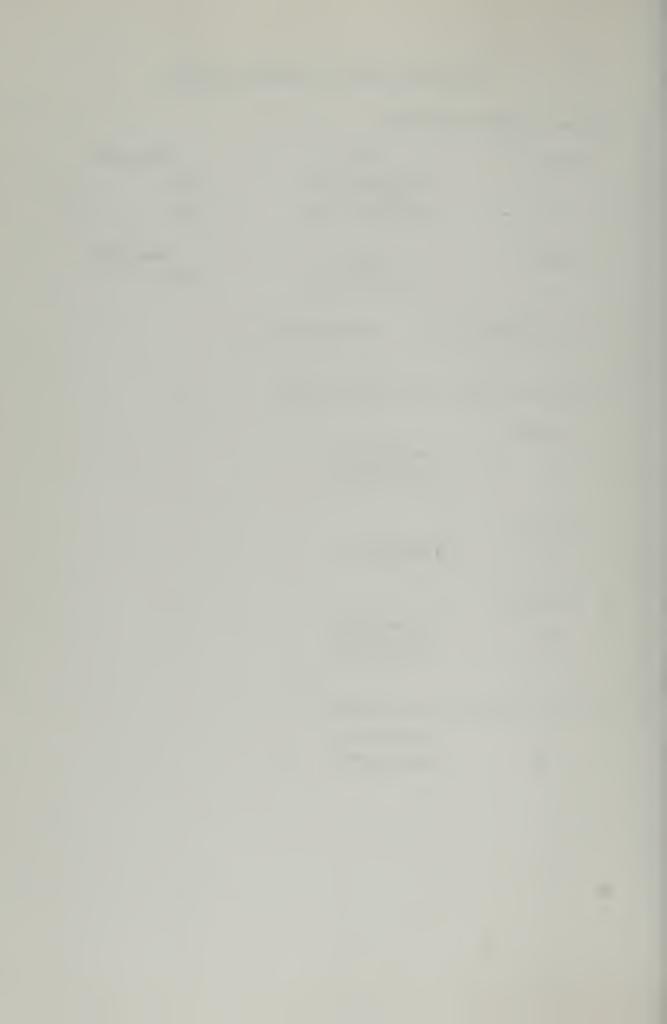
2 9.99122047

C VECTOR

1	14.99480128
2	1.000000000

INITIAL CONDITION (G) VECTOR

1	14.99876932	
2	584.3008098	



SYSTEM TO BE IDENTIFIED

SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY	
1	-1.0000000000	-1.0000000000	J
2	-1.0000000000	1.0000000000	J
3	-50.0000000000	0.0	J
ZEROES	REAL	IMAGINARY	
1	-5.000000000	0.0	J
2	-20.000000000	0.0	J
TRIADO MIAO	ANT - E DOODOODOO		

GAIN CONSTANT = 5.0000000000

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR	
1	100.0000000000
2	102.0000000000
3	52.0000000000
B VECTOR	
3	5.0000000000
C VECTOR	
1	100.00000000000
2	25.0000000000
3	1.0000060000

1	2.0000000000
2	1.0000000000
2	6.0



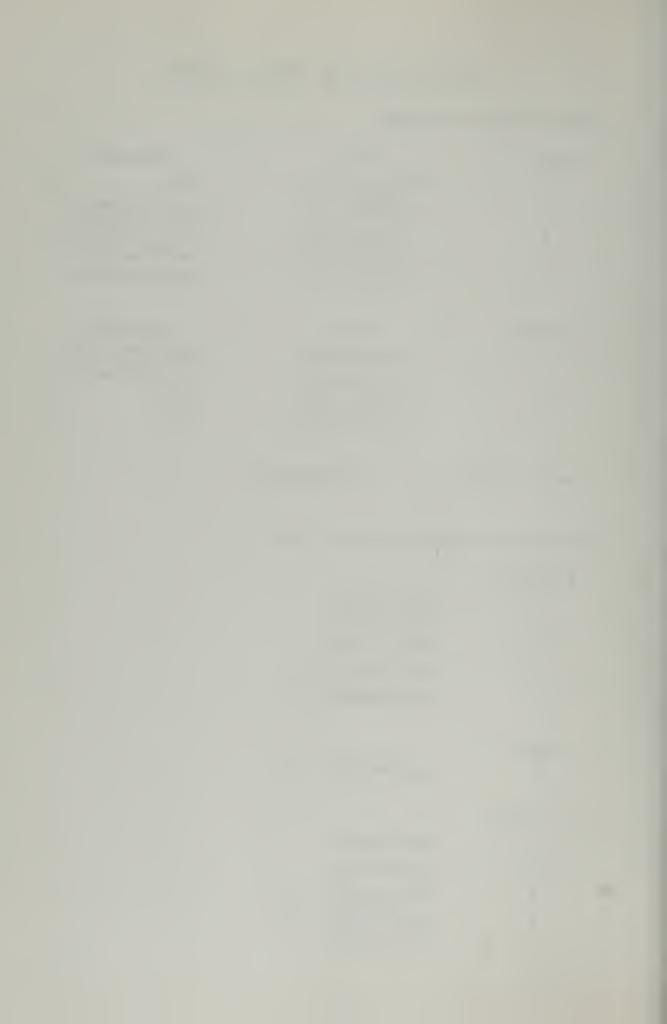
SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY
1	-49.9911920718	0.0 J
2	1.1558667717	-23.4137133914 J
3	1.1558667717	23.4137133914 J
4	-0.9997223909	-C.9974653718 J
5	-0.9997223909	0.9974653718 J
ZERDES	REAL	IMAGINARY .
1	1.2179838023	-23.3753514222 J
2	1.2179838023	23.3753514222 J
3	-19.8075068598	0.0 J
4	-5.1152815779	0.0 J

GAIN CONSTANT = 4.9560489655

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR	
1	54789.78240
2	55794.37204
3	28434.85333
4	531.2985138
5	49.67890331
B VECTOR	
5	4.956048965
C VECTOR	
1	55512.80354
2	13408.14537
3	588.5004083
4	22.48682083
5	1.000000000



INITIAL CONDITION (G) VECTOR

1 225.0001273

2 10975.84397

3 117558.8284

4 6270060.545

5 11259586.57

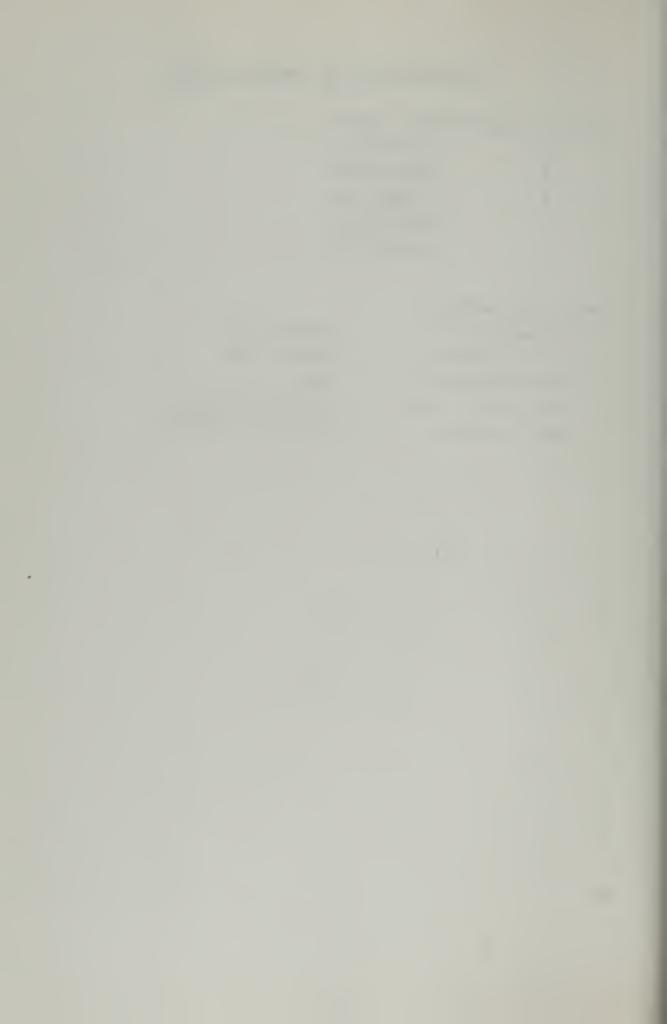
PROGRAM PARAMETERS

RECORD LEGNTH = 1.20000 SEC

SAMPLING PERIOD = 0.20000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 4 DECIMAL PLACES



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY
1	-4 9•9862042435	0.0 J
2	-0.9995303000	-1.0007669686 J
3	-0.9995303000	1.0007669686 J
ZEROES	REAL	IMAGINARY
1	-4.9622611725	0.0 J
2	-20.2677062457	0•0 J.

GAIN CONSTANT = 4.9685173035

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR	
1	100.0021676
2	101.9260468
3	51.98526484
B VECTOR	
3	4.968517303
C VECTOR	
1	100.5736518
2	25.22996742
3	1.000000000

INITIAL CONDITION (G) VECTOR 1 225.0005829

2 11494.69251 3 20485.03332



SYSTEM TO BE IDENTIFIED

SYSTEM TRANSFER FUNCTION

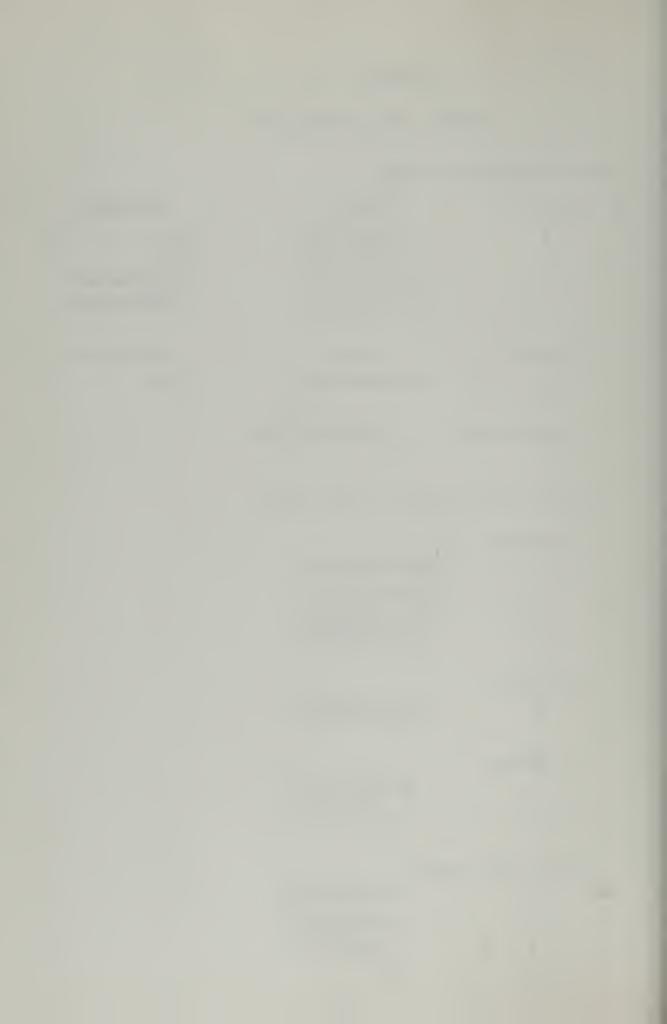
POLES	REAL	IMAGINARY	
1	-3.000000000	0.0	J
2	-8.000000000	0.0	J
3	-40.000000000	-5.0000000000	J
4	-40.000000000	5.0000000000	J
Z EROES	REAL	IMAGINARY	
1	-20.0006000000	0.0	J

SYSTEM STATE VARIABLES (PHASE FORM)

GAIN CONSTANT = 160.0000000000

A VECTOR	
1	39000.000000000
2	19795.0000000000
3	2529.00000000000
4	91.0000000000
B VECTOR 4	160.0000000000
C VECTOR	
1	20.0000000000
2	1.0000000000

1	3.0000000000
2	2.0000000000
3	1.00000000000
4	0.0



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY	
1	-40.0576848888	-4.864893972	7 J
2	-40.0576848888	4.864893972	7 J
3	-8.0002609098	0.0	J
4	-1.4530133153	0.0	J
5	-2.9998483817	0.0	J
ZEROES	REAL	 IMAGINARY	
1	-1.4492282436	0.0	J
2	-20.3685768935	0.0	J

GAIN CONSTANT = 157.7509613037

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR		
1	56781.06779	
2	67897.28681	
3	23515.35114	
4	2665.954710	
5	92.56849238	
B VECTOR		
5	157.7509613	
C VECTOR		•
1	29.51871692	
2	21.81780514	
3	1.000000000	



INITIAL CONDITION (G) VECTOR

1 62.00108720

2 5780.206688

3 169108.2337

4 1409764.537

5 1708815.026

PROGRAM PARAMETERS

RECORD LEGNTH = 1.48842 SEC

SAMPLING PERIOD = 0.24807 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 5 DECIMAL PLACES



SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY	
1	-40.0840542053	-4.7409985601	J
2	-40.0840542053	4.7409985601	J
3	-3.0000451598	0.0	J
4	-7.9997100927	0.0	J
ZEROES	REAL	IMAGINARY	
1	-19.9584019372	0.0	J.
4 ZEROES	-7.9997100927 REAL	0.0 IMAGINARY	Ĭ

GAIN CONSTANT = 160.7285003662

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR		
1	39100.17487	
2	19844.88825	
3	2535.037532	
4	91.16786366	
B VECTOR		
4	160.7285004	
C VECTOR		
1	19.95840194	
2	1.000000000	
		-

INITIAL CONDITION (G) VECTOR 1 62.00114277 2 5693.349573 3 160935.4623 4 1176687.962



SYSTEM TO BE IDENTIFIED

SYSTEM TRANSFER FUNCTION

POLES	REAL	IMAGINARY	
1	-1.000000000	0.0	J
2	-5.000000000	0.0	J

IMAGINARY

REAL

SYSTEM STATE VARIABLES (PHASE FORM)

GAIN CONSTANT = 10.0000000000

A VECTOR	
1	5.0000000000

2 6.0000000000

B VECTOR

ZEROES

2 10.000000000

C VECTOR

1.0000000000

INITIAL STATE VECTOR

1 1.00000000000



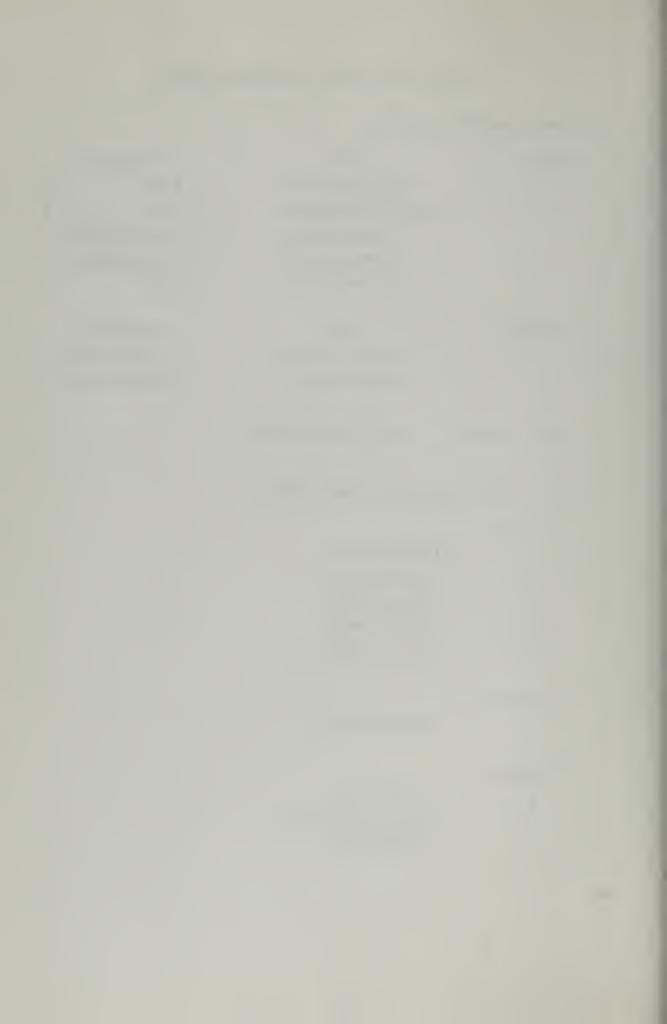
SYSTEM TRANSFER FUNCTION

_a P	OLES	REAL	IMAGINARY	
	1	-5.0110029817	0.0	J
	2	-2874.4029427054	0.0	J
	3	0.0094513173	-2.4186034399	J
	4	0.0094513173	2.4186034399	J
	5	-0.9999781363	0.0	J
Z	EROES	REAL	IMAGINARY	
	1	0.0096691675	-2.4198473522	J
	2	0.0096691675	2.4198473522	J

GAIN CONSTANT = 28777.0156250000

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR	
1	84255.60063
2	100828.6134
3	30926.28200 .
4	17234.39503
5	2880.395021
B VECTOR	
5	28777.01563
C VECTOR	•
1	5.855754701
2	-0.1933833505D-01
3	1.000000000



INITIAL CONDITION (G) VECTOR

1 1.450317645

2 2876.228032

3 17252.85740

4 16462.65716

5 101232.6568

PROGRAM PARAMETERS

RECORD LEGNTH = 12.00000 SEC

SAMPLING PERIOD = 2.00000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 5 DECIMAL PLACES



SYSTEM TRANSFER FUNCTION

POLES REAL IMAGINARY

1 -0.9999961462 0.0 J

2 -5.0000869786 0.0 J

ZEROES REAL IMAGINARY

GAIN CONSTANT = 10.0001306534

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR

1 5.000067709

2 6.000083125

B VECTOR

2 10.00013065

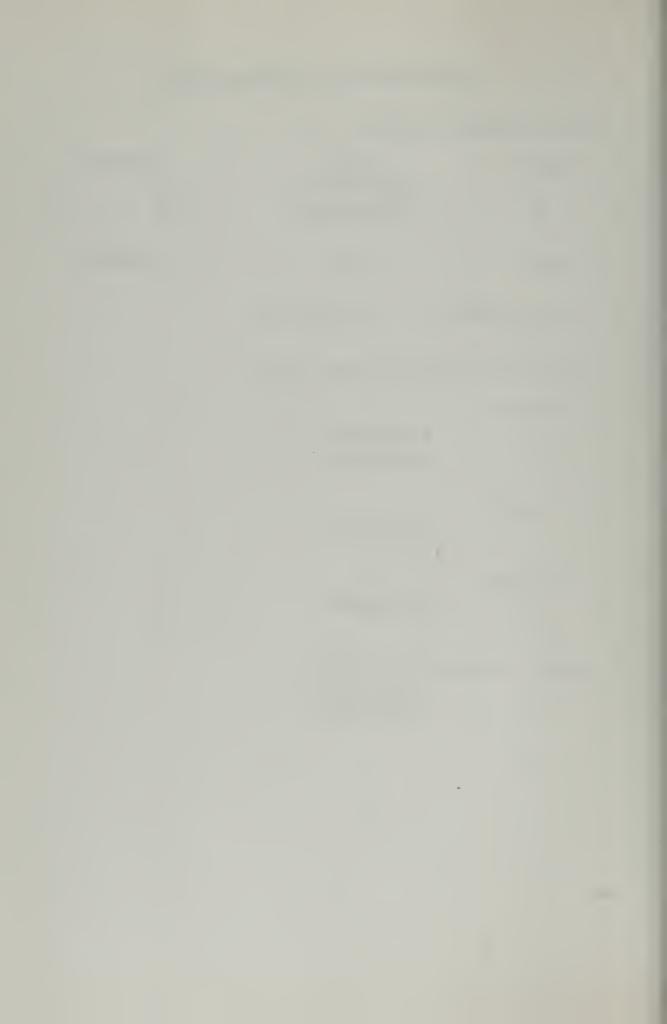
C VECTOR

1 1.000000000

INITIAL CONDITION (G) VECTOR

1 1.000025784

2 6.000036891



SYSTEM TO BE IDENTIFIED

SYSTEM TRANSFER FUNCTION

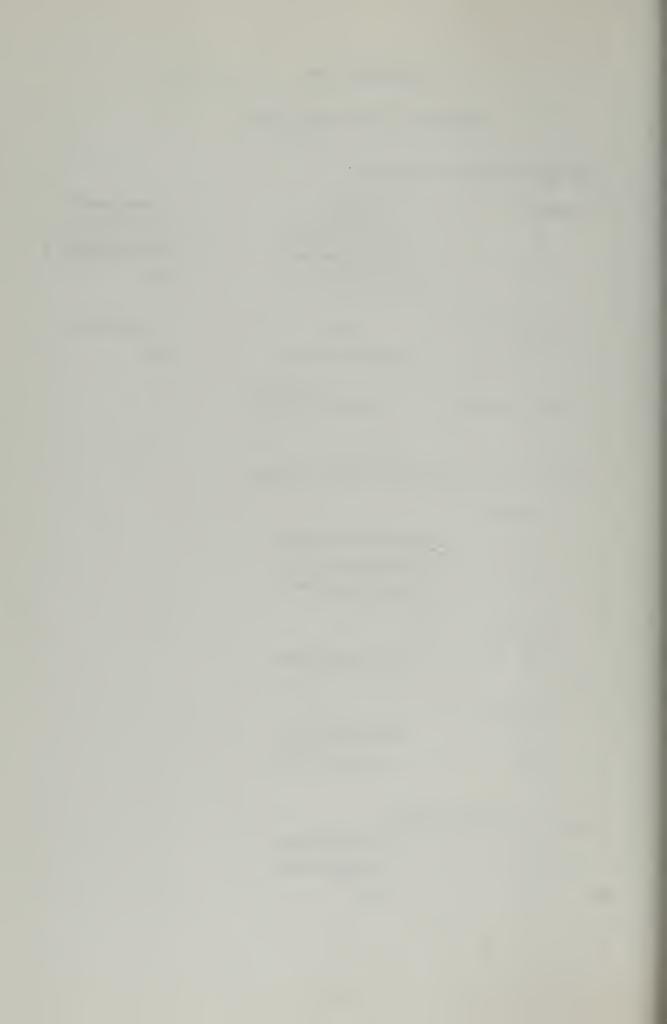
POLES	REAL	IMAGINARY	
1	-5.0000000000	-5.0000000000	J
2	-5.000000000	5.0000000000	J
3	-100.000000000	0.0	J
ZEROES	REAL	IMAGINARY	
_ 1	-20.000000000	0.0	J

GAIN CONSTANT = 100.000000000

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR	
1	5000.0000000000
2	1050.0000000000
3	110.0000000000
B VECTOR 3	100.000000000
C VECTOR	
1	20.0000000000
2	1.000000000000

1	2.0000000000
2	1.0000000000
3	0.0



IDENTIFICATION OF UNKNOWN SYSTEM

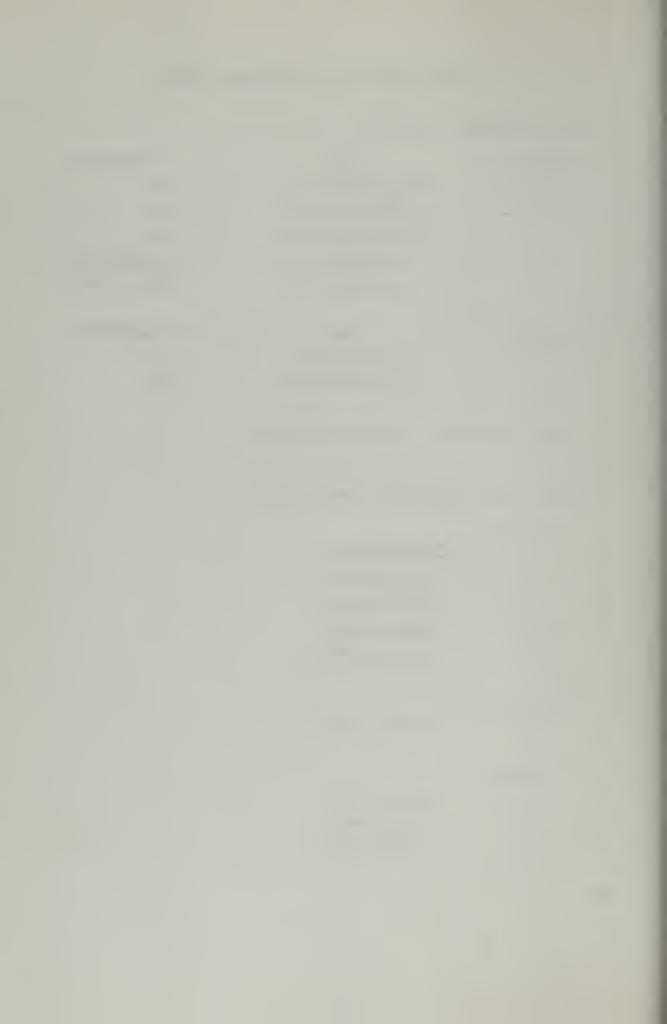
SYSTEM TRANSFER FUNCTION

POLES		R EAL	IMAGINARY	
	1	3533.8075084749	0.0	J
	2	-25,9614103219	0.0	J
	3	-100.0395044818	0.0	J
	4	-4.9998977161	-5.0000893933	J
	5	-4.9998977161	5.0000893933	J
	ZEROES	REAL	IMAGINARY	
	1	-19.5630088534	C.O	J
	2	-26.9189879242	0.0	J

GAIN CONSTANT =-348534.3750000000

SYSTEM STATE VARIABLES (PHASE FORM)

A VECTOR	
1	-458893162.9
2	-113910225.2
3	-13774844.37
4	-476693.1811
5	-3397.806798
B VECTOR 5	-348534.3750
C VECTOR	
1	526.6163991
2	46.48199678
3	1.000000000



IDENTIFICATION OF UNKNOWN SYSTEM

INITIAL CONDITION (G) VECTOR

1 40.97649863

2 -139289.1252

3 -19623611.76

-535**7**513**77**•4

5 -3138782043.

PROGRAM PARAMETERS

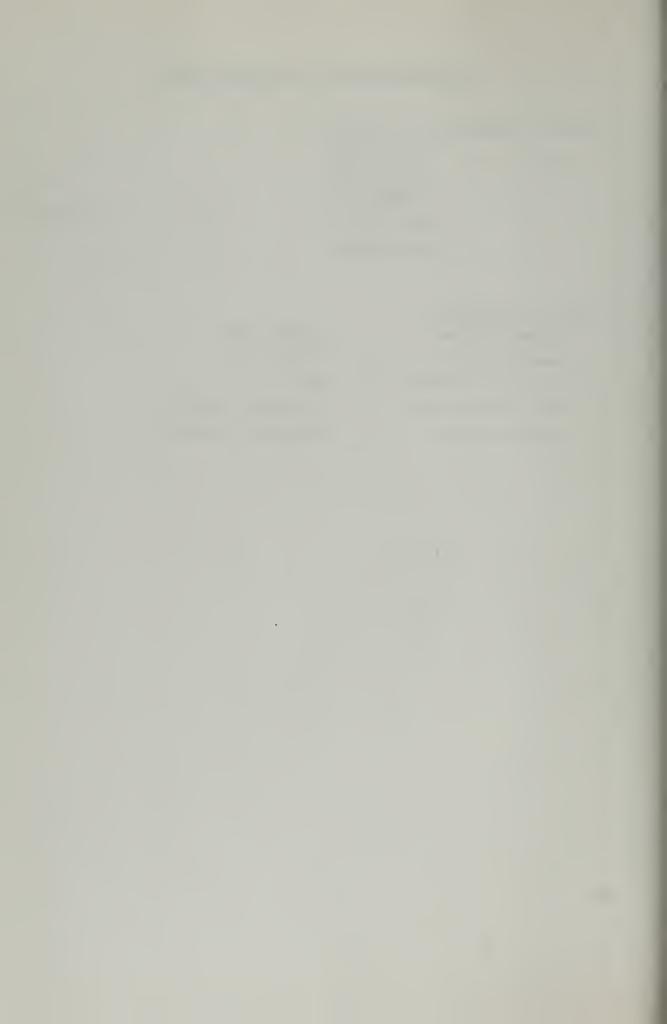
RECORD LEGNTH = 0.60000 SEC

SAMPLING PERIOD = 0.10000 MSC

EQUATIONS FORMED = 120

PRECISION OF DATA = 5 DECIMAL PLACES

INPUT FUNCTION = PIECEWISE CONSTANT



IDENTIFICATION OF UNKNOWN SYSTEM

SYSTEM TRANSFER FUNCTION

ARY
J
4476 J
4476 J
ARY
J

GAIN CONSTANT = 100.0136718750

SYSTEM STATE VARIABLES (PHASE FORM)

•	3 4		-	~		2
Α	v	-	C-	1	t 1	ĸ
_		_	\sim	3	\mathbf{c}	1 /

1	4999.830647
2	1049.959363
3	109.9959607

B VECTOR

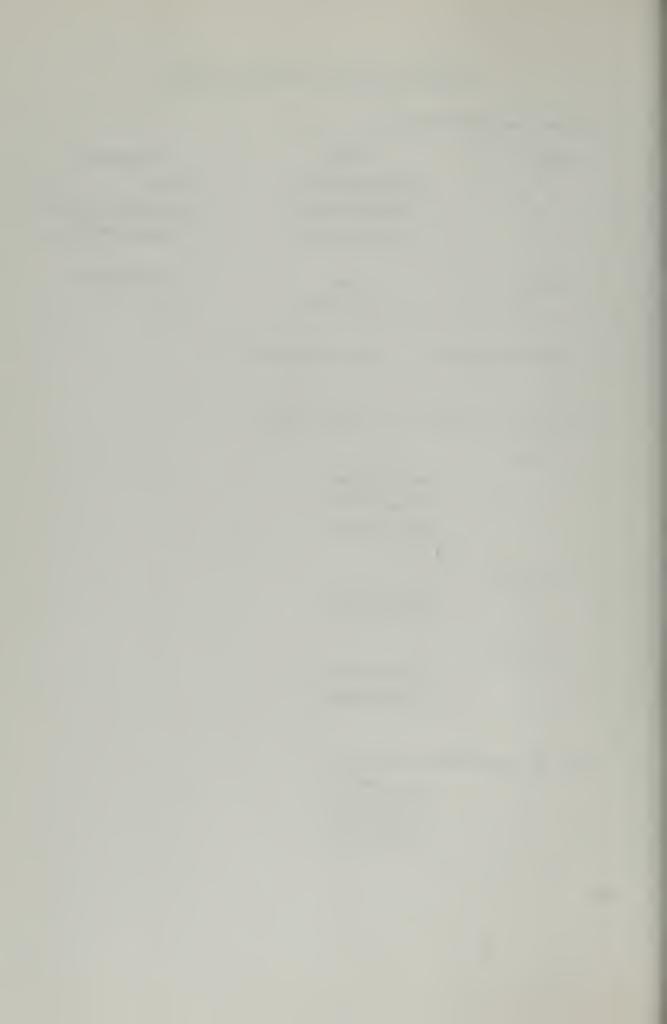
3 100.0136719

C VECTOR

1	19.99608414
2	1.000000000

INITIAL CONDITION (G) VECTOR

1	40.99999105
2	4529.827485
3	34198.63350



IV. CONCLUSIONS

The method of multiple integrations is a practical and flexible method for identifying lumped parameter, linear, time invariant systems. Complete and accurate identifications can be made on the basis of arbitrary input-output records taken over a short time interval and accurate to only three or four significant figures. The computational requirements of the method are not excessive. The procedure can be implemented by relying entirely on subroutines which are available in most computer center libraries.

At present the technique is limited to comparatively low order systems. When the input-output records are good to three or four significant digits the method will be capable of identifying systems up to about fifth order. This limitation is due primarily to the algorithm used to solve the overdetermined set of linear equations. As better algorithms become available it will be possible to identify higher order systems.

The accuracy of an identification is usually comparable to the number of significant digits in the inputoutput data. Accuracy depends to a lesser extent on the sampling rate, the nature of the input function driving the system, and the order of the system.

There are several areas where additional research might prove fruitful. Since the input-output records must



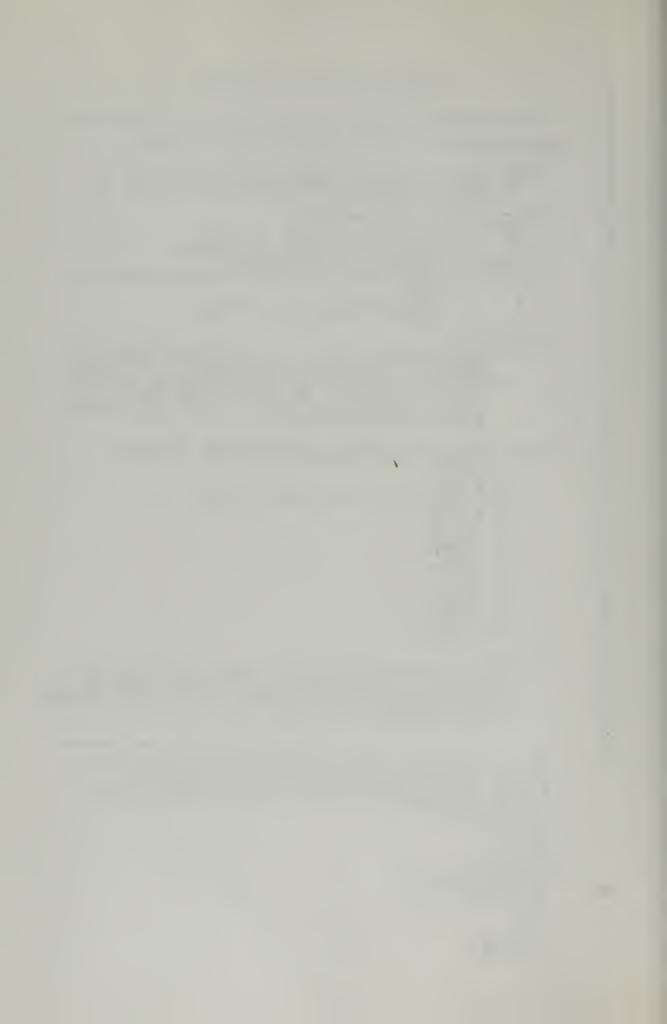
be in sampled data form for the computer it might be profitable to reformulate the identification procedure from a sampled system standpoint. Standard techniques could be used to convert the sampled system representation obtained from the identification to a continuous system representation. The ability to identify sampled systems would be a worthwhile extension of this method.

The method of multiple integrations may prove very useful as a tool for approximating high order systems with low order systems. Research could be done to determine the quality of the approximations obtained using this method.



DIGITAL COMPUTER PROGRAMS

```
MAIN PROGRAM - LINEAR SYSTEM IDENTIFICATION
PURPOSE
TO IDENTIFY LINEAR TIME INVARIANT SYSTEMS ON
THE BASIS OF INPUT-OUTPUT RECORDS
DESCRIPTION OF PARAMETERS
       INPUT
NP
NZ
                               ESTIMATED NUMBER OF POLES
ESTIMATED NUMBER OF ZEROES
NUMBER OF DATA POINTS
        KPTMAX
        IPTS
                               DATA POINTS
                                                             INTEGRATED PER LINEAR EQ.
                               INPUT AMPLITUDE AT TIME OUTPUT AMPLITUDE AT TIME
        RC
REMARKS (1)
                  OUTPUT WILL CONSIST OF A TRANSFER FUNCTION AND STATE VARIABLE REPRESENTATION OF SYSTEM PROGRAM IS PRESENTLY CONFIGURED TO IDENTIFY SYSTEMS SIMULATED BY SUBROUTINE SYSTEM. IF IT IS DESIRED TO IDENTIFY A PHYSICAL SYSTEM REPLACE SUBROUTINE SYSTEM CALLS WITH APPROPRIATE READ STATEMENTS.
        (2)
SUBROUTINES AND FUNCTION SUBPROGRAMS WHEN IDENTIFYING PHYSICAL SYSTEMS (1) DLLSQ (2) RTPLSB
                                                                                                REQUIRED
        WHEN IDENTIFYING SIMULATED SYSTEMS
                  DLLSQ
RTPLSB
SYSTEM
EXPAND
        (1)
(2)
(3)
(4)
(5)
(6)
(7)
(8)
(9)
                   SUMM
                  DIFF
PROD
GAUSS3
ARRAY
MINV
           10
                     ROUND
METHOD
        MULTIPLE INTEGRALS OF THE INPUT AND OUTPUT DAY
ARE USED TO FORMULATE A SET OF OVERDETERMINED
LINEAR EQUATIONS. THESE EQUATIONS ARE THEN SO
FOR THE UNKNOWN MODEL PARAMETERS USING THE ME
OF LEAST SQUARES.
                                                                                                                        DATA
                                                                                                                         SOL VED
                                                                                                                        METHOD
 . . . . . . . . . . . . . . . . . . .
REAL*8 TO,TN(11),RO(11),RN(11),CO(11),CN(11),DT2
REAL*8 A(26C0),B(260),X(26),AUX(52),CONV(9)
REAL*8 AP(10),AZ(10),PRA(9),PIA(9),ZRA(8),ZIA(8)
INTEGER IPIV(26)
KPTMAX=5002
IPTS=50
NP=4
NZ-NP=1
 NZ=NP-1
CCNTINUE
MEQS=KPTMAX/IPTS
TO=-1.00
EPS=10.0**(-35)
 K=1
M=0
 NP1=NP+1
NP2=NP+2
```



```
NZ1 = NZ + 1
          N=NP+NZ+1
          NI = N + NP
CCC
          SET CUMULATIVE INTEGRAL VALUES TO ZERO
     DO 10 I=2, NP2
RO(I)=0.00
10 CO(I)=0.00
CCC
          READ IN INITIAL DATA POINT
                                                          (T,R,C)
          CALL SYSTEM(TO, RO(1), CO(1))
C
          TOFF=TO
     15 K=K+1
CCC
          READ IN NEW DATA POINT
                                                     (T,R,C)
          CALL SYSTEM(TN(1), RN(1), CN(1))
CCC
          UPDATE MULTIPLE INTEGRATIONS
          DT2=(TN(1)-T0)*0.5
DO 20 INT=1,NP1
RN(INT+1)=(RO(INT)+RN(INT))*DT2+RO(INT+1)
CN(INT+1)=(CO(INT)+CN(INT))*DT2+CO(INT+1)
CCC
           FORM A LINEAR EQUATION
           IF (K.NE.(K/IPTS)*IPTS) GO TO 35
          M = M + 1
           B(M) = CN(2)
          TN(2)=(TN(1)-TOFF)
DO 25 I=1,NP
IA=(NP-I)*MEQS+M
          ID=(NP-1)*MEQS+M
IC=(N+I-1)*MEQS+M
TN(I+2)=TN(I+1)*TN(2)/FLOAT(I+1)
A(IA)=-CN(I+2)
A(IC)=TN(I+1)
DO 30 I=1,NZ1
IA=(NP+I-1)*MEQS+M
IRN=NP+3-I
A(IA)-PN(IDN)
          A(IA) = RN(IRN)
     30
CCC
           RESET OLD VALUES
          TO=TN(1)
DO 40 I=1, NP2
RO(I)=RN(I)
CO(I)=CN(I)
IF (M.LT.MEQS) GO TO 15
     35
000 000
           SOLVE FOR PARAMETERS BY METHOD OF LEAST SQUARES
           CALL DLLSQ(A,B,M,NI,1,X,IPIV,EPS,IER,AUX)
           CALCULATE POLES
           AP(1)=1.00
           DO 45 I=1,NP

J=NP+2-I

AP(J)=X(I)

CALL RTPLSB(NP,AP,PRA,PIA,CONV,IERPZ)
           CALCULATE ZEROES
           GAINI=X(N)
DO 50 I=NP1,N
X(I)=X(I)/X(N)
DO 55 I=1,NZ1
      50
           J=N+1-I
```



```
AZ(I)=X(J)
IF (NZ.EQ.O) GO TO 60
CALL RTPLSB(NZ,AZ,ZRA,ZIA,CONV,IERPZ)
CONTINUE
          55
                                OUTPUT
                             WRITE(6,915)
WRITE(6,916)
WRITE(6,917)
WRITE(6,917)
WRITE(6,919)
WRITE(6,920)
WRITE(6,920)
WRITE(6,901)
WRITE(6,901)
WRITE(6,902)
DO GONTINUE
WRITE(6,903)
CONTINUE
WRITE(6,903)
CONTINUE
WRITE(6,903)
CONTINUE
WRITE(6,906)
WRITE(6,906)
WRITE(6,907)
DO 80 I=1,NP
I=NP+2-I
WRITE(6,907)
DO 80 I=1,NP
I=NP+2-I
WRITE(6,910)
WRITE(6,910)
WRITE(6,910)
WRITE(6,910)
WRITE(6,910)
WRITE(6,910)
WRITE(6,910)
WRITE(6,910)
WRITE(6,910)
URRITE(6,913)
DO 90 I=1,NP
                                                                                                                                   IPTS
EPS
AUX(1)
                                                                                                                                    IER
(IPIV(I),I=1,NI)
                                                                                                                                    I, PRA(I), PIA(I)
                                                                                                                                    GO TO 75
                                                                                                                                    I, ZRA(I), ZIA(I)
                                                                                                                                    GAINI
                                                                                                                                   I, AP(II)
        80
                                                                                                                                  NP . GAINI
                                                                                                                                I,AZ(II)
        85
                              II=N+I
WRITE(6,910) I,X(II)
CONTINUE
GO TO 5
                   WRITE(6,910) 1,x(11)
CONTINUE
GO TO 5
FORMAT(1H1,///,25X,'IDENTIFICATION OF UNKNOWN SYSTEM')
FORMAT(//,15X,'SYSTEM TRANSFER FUNCTION')
FORMAT(//,15X,'POLES',12X,'REAL',13X,'IMAGINARY',/)
FORMAT(//,15X,'ZEROES',11X,'REAL',13X,'IMAGINARY',/)
FORMAT(//,15X,'GAIN CONSTANT =',G15.8,/)
FORMAT(//,15X,'GAIN CONSTANT =',G15.8,/)
FORMAT(//,15X,'A VECTOR',/)
FORMAT(//,15X,'A VECTOR',/)
FORMAT(//,15X,'B VECTOR',/)
FORMAT(//,15X,'B VECTOR',/)
FORMAT(//,15X,'G15.8,/)
FORMAT(//,15X,'G15.8,/)
FORMAT(17X,12,7X,G15.8,/)
FORMAT(11X,12,7X,G15.8,/)
FORMAT(11X,12,7X,G15.8,/)
FORMAT(1/,5X,'LER',15,5X,'EPS',E15.8,5X,'AUX',
1E16.8,/,9X,1118)
2 FORMAT(//,75X,4(E20.9,5X),//,5X,4(E20.9,5X),//)
3 FORMAT(//,7/,12X,'PROGRAM PARAMETERS',/)
6 FORMAT(15X,'INIMBER OF DATA POINTS =',16,/)
7 FORMAT(15X,'NUMBER OF DATA POINTS =',16,/)
8 FORMAT(15X,'NUMBER OF DATA POINTS =',16,/)
9 FORMAT(15X,'NUMBER OF DATA POINTS =',16,/)
9 FORMAT(15X,'BAS ERROR =',E16.8,/)
9 FORMAT(15X,'EPS =',E16.8,/)
9 FORMAT(15X,'IPIV(I) =',15(I2,1X),/)
8 TOP
END
         90
900
9001
9002
9003
9005
9007
9008
9009
910
911
912
913
914
915
916
917
918
920
921
```



```
SUBROUTINE SYSTEM
     PURPOSE
GENERATES INPUT-OUTPUT RECORDS FOR IDENTIFICATION PROGRAM BY SIMULATING A SYSTEM DESCRIBED BY A TRANSFER FUNCTION THAT IS READ IN
      USAGE
             CALL
                        SYSTEM(T,R,C)
      DESCRIPTION OF PARAMETERS
             INPUT
NP
                                  NUMBER OF POLES
THE VECTOR OF POLES
NUMBER OF ZEROES
THE VECTOR OF ZEROES
THE GAIN CONSTANT
             P(I)
             NZ
Z(I)
GAIN
             OUTPUT
                                  TIME
INPUT AMPLITUDE AT TIME T
OUTPUT AMPLITUDE AT TIME T
             Ŕ
      REMARKS
                      INPUT FUNCTION MAY BE CHANGED BY CHANGING ONE CARD IN PROGRAM R AND C ARE ROUNDED TO NA DIGITS ALL FLOATING POINT VARIABLES ARE DOUBLE PRECISION (REAL*8).
             (1)
             (2)
(3)
                                                                                                    DECLARED
      SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
             (1)
(23)
(4)
(5)
(6)
(7)
                       SUMM
                       DIFF
                       PROD
EXPAND
GAUSS3
ARRAY
                       MINV
             (7)
                       ROUND
      METHOD
             TRANSFER FUNCTION IS CONVERTED TO STATE VARIABLE REPRESENTATION AND INTEGRATED USING TRAPEZOIDAL INTEGRATION
      SUBROUTINE SYSTEM(T,R,C)
      REAL*8 AA(10),CC(10),A(9,9),B(9,1),XXX(9,9)
REAL*8 PHI(9,9),DEL(9,1),XX(9,1),UU(1,1)
REAL*8 T,R,C,U,DT,AI(9,9),ZZZ(9,9)
COMPLEX*16 P(9),Z(8)
       IF (T.GE.O.OU) GO TO 55
       INPUT POLES, ZEROES, AND GAIN CONSTANT
      READ(5,899,END=999) NP

READ(5,898) (P(I),I=1,NP)

CALL EXPAND (NP,P,AA)

DO 5 I=1,NP

CC(I)=0.00

CONTINUE

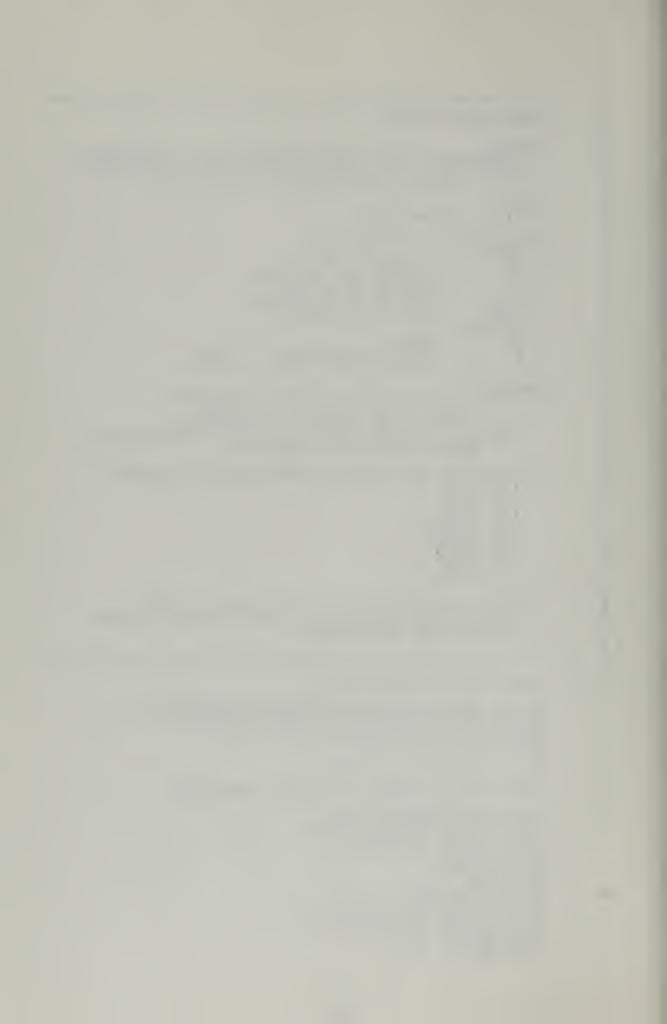
READ(5,899) NZ

IF (NZ.EQ.0) GO TO 10

READ(5,898) (Z(I),I=1,NZ)

CALL EXPAND (NZ,Z,CC)

CONTINUE
      CALL EXP
10
       READ(5,898)
                                  GAIN
```



```
CCC
               SET NA
              NA=3
CCC
               FIND SHORTEST TIME CONSTANT AND CALCULATE DT
             DT=0.00
DO 15 I=1,NP
IF (DT.LT.CDABS(P(I))) DT=CDABS(P(I))
CONTINUE
IF (NZ.EQ.C) GO TO 25
DO 20 I=1,NZ
IF (DT.LT.CDABS(Z(I))) DT=CDABS(Z(I))
CONTINUE
DT=1.0/(DT*190.0)
IF (DT.EQ.0.GO) DT=0.00C01
       15
000
               FORM A, B, AND C MATRICES
              DO 35 I=1,NP

DO 30 J=1,NP

AI(I,J)=0.00

A(I,J)=0.00

CONTINUE

B(I,1)=0.00

CONTINUE

DO 40 I=2,NP

AI(I,I)=1.00

A(I-1,I)=DT/2.00

A(NP,I)=-AA(I)*DT/2.00

CONTINUE

AI(I,1)=1.00

A(NP,I)=-AA(1)*DT/2.00

B(NP,I)=GAIN*DT/2.00

CC(NZ+1)=1.00
       30
        35
       40
CCC
               CALCULATE PHI MATRIX
                          DIFF(AI,A,ZZZ,NP,NP)
GAUSS3(NP,EPSS,ZZZ,XXX,KER,9)
SUMM(AI,A,ZZZ,NP,NP)
PROD(XXX,ZZZ,PHI,NP,NP,NP)
               CALL
               CALL
CCC
               CALCULATE DEL MATRIX
               CALL PROD(XXX, B, DEL, NP, NP, 1)
000
               DEFINE INITIAL CONDITIONS
               K=1
               DO 45 I=1,NP

XX(I,1)=FLOAT(NP-I)

CONTINUE

UU(1,1)=0.00

T=0.00

R=0.00
               C=0.00
C=0.00
D0 50 I=1, NP
C=CC(I)*XX(I,1)+C
CONTINUE
G0 T0 65
        50
000
                CALCULATE NEW DATA POINT (T,R,C)
               CONTINUE
T=DT*FLOAT(K)
 CCC
                INPUT FUNCTION
                           +0.1*FLOAT(K/53)+FLOAT(K/403)+FLOAT(K/603)
                U=
 C
```



```
UU(1,1)=UU(1,1)+U
CALL PROD(PHI,XX,XXX,NP,NP,1)
CALL PROD(DEL,UU,ZZZ,NP,1,1)
C=0.00
DO 60 I=1,NP
XX(I,1)=XXX(I,1)+ZZZ(I,1)
C=CC(I)*XX(I,1)+C
CCNTINUE
                 60
                                  R=U
                                  ROUND OFF R AND
                                 CALL ROUND(R,NA)
CALL ROUND(C,NA)
C
                                 UU(1,1)=U
K=K+1
C
                                 RETURN
CCC
                                 OUTPUT
                               OUTPUT

CONTINUE

WRITE(6,900)

WRITE(6,902)

DO 70 I=1,NP

WRITE(6,903)

CONTINUE

WRITE(6,903)

CONTINUE

WRITE(6,903)

CONTINUE

WRITE(6,903)

CONTINUE

WRITE(6,906)

WRITE(6,906)

WRITE(6,906)

WRITE(6,907)

DO 85 I=1,NP

WRITE(6,909)

WRITE(6,910)

CONTINUE

WRITE(6,910)

CONTINUE

WRITE(6,910)

WRITE(6,910)

WRITE(6,910)

WRITE(6,910)

WRITE(6,910)

WRITE(6,910)

WRITE(6,910)

WRITE(6,910)

WRITE(6,910)

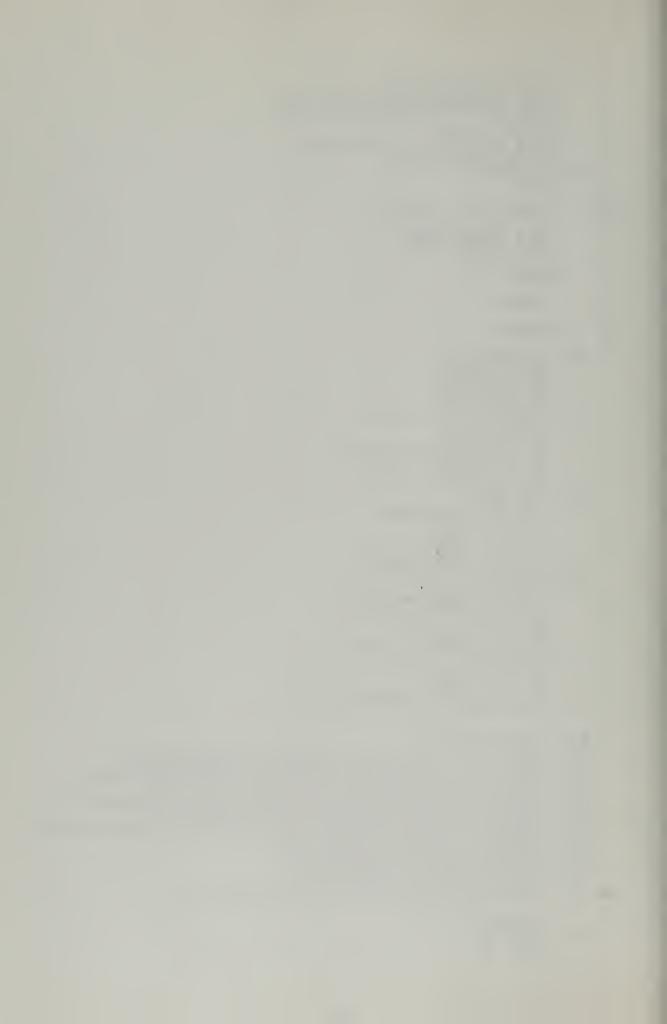
OONTINUE

WRITE(6,910)

WRITE(6,910)

OONTINUE

WRITE(6,910)
                 65
                                                                                                         I, P(I)
                 70
                                                                                                        GO TO 80
                                                                                                        I, Z(I)
                                                                                                        GAIN
                                                                                                         I,AA(I)
                85
                                                                                                       NP, GAIN
                                                                                                       I,CC(I)
                90
                                WRITE(6,913)
DO 95 I=1,NP
WRITE(6,910)
CONTINUE
                                                                                                    I, XX(I,1)
                 95
                              FORMAT(2F10.5)
FORMAT(11)
FORMAT(1H1,///,25X,'SYSTEM TO BE IDENTIFIED')
FORMAT(//,12X,'SYSTEM TRANSFER FUNCTION')
FORMAT(//,15X,'POLES',12X,'REAL',13X,'IMAGINARY',/)
FORMAT(17X,12,7X,F14.7,6X,F14.7,1X,'J',/)
FORMAT(//,15X,'ZEROES',11X,'REAL',13X,'IMAGINARY',/)
FORMAT(//,15X,'GAIN CONSTANT =',F14.7,/)
FORMAT(//,15X,'SYSTEM STATE VARIABLES (PHASE FORM)
FORMAT(//,15X,'A VECTOR',/)
FORMAT(//,15X,'B VECTOR',/)
FORMAT(//,15X,'C VECTOR',/)
FORMAT(//,15X,'INITIAL STATE VECTOR',/)
FORMAT(//,15X,'INITIAL STATE VECTOR',/)
          898
          899
900
          90123
90023
9005
9005
9006
9008
9009
9103
                                                                                                                                                                                                                                                            (PHASE FORM) 1)
                                RETURN
CONTINUE
STOP
END
          999
```



SUBROUTINE DLLSO

PURPOSE TO SOLVE OLVE LINEAR LEAST SQUARES PROBLEMS, I.E. TO MIZE THE EUCLIDEAN NORM OF B-A*X, WHERE A IS BY N MATRIX WITH M NOT LESS THAN N. IN THE IAL CASE M=N SYSTEMS OF LINEAR EQUATIONS MAY MINIMIZE A M BY N SPECIAL SOLVED.

USAGE CALL DLLSQ(A,B,M,N,L,X,IPIV,EPS,IER,AUX)

DESCRIPTION

M BY N MATRIX

B RIGHT HAND SIDE

AND B

OF PARAMETERS
DOUBLE PRECISION M BY N MAT
(DESTROYED).
DOUBLE PRECISION M BY L RIG
MATRIX (DESTROYED).
ROW NUMBER OF MATRICES A AN
COLUMN NUMBER OF MATRIX A, N ROW NUMBER OF

MATRIX X
COLUMN NUMBER OF MATRICES B AND X
DOUBLE PRECISION N BY L SOLUTION MATRIX
INTEGER OUTPUT VECTOR OF DIMENSION N
WHICH CONTAINS INFORMATION ON COLUMN
INTERCHANGES IN MATRIX A.
SINGLE PRECISION INPUT PARAMETER WHICH
SPECIFIES A RELATIVE TOLERANCE FOR
DETERMINATION OF RANK OF A. X IPIV

EPS

DETERMINATION OF RANK OF A.
A RESULTING ERROR PARAMETER
A DOUBLE PRECISION AUXILIARY STOL
ARRAY OF DIMENSION MAX(2*N,L). OF
FIRST L LOCATIONS OF AUX CONTAIN
RESULTING LEAST SQUARES. IER AUX STORAGE ON RETURN

REMARKS

(1)

(2)

NO ACTION BESIDES ERROR MESSAGE IER=-2 I CASE M LESS THAN NO NO ACTION BESIDES ERROR MESSAGE IER=-1 I CASE OF A ZERO MATRIX AO IF RANK K OF MATRIX A IS FOUND TO BE LESTHAN N BUT GREATER THAN COUNTY THE PROCEDURE RETURNS WITH ERROR CODE IER=K INTO CALLI PROGRAMO THE LAST NOK ELEMENTS OF VECTOR DENOTE THE USELESS COLUMNS IN MATRIX AO IF THE PROCEDURE WAS SUCCESSFUL, ERROR PARAMETER IER IS SET TO ZEROO (3) CALLING ENTS OF VECTOR IPIV

(4)

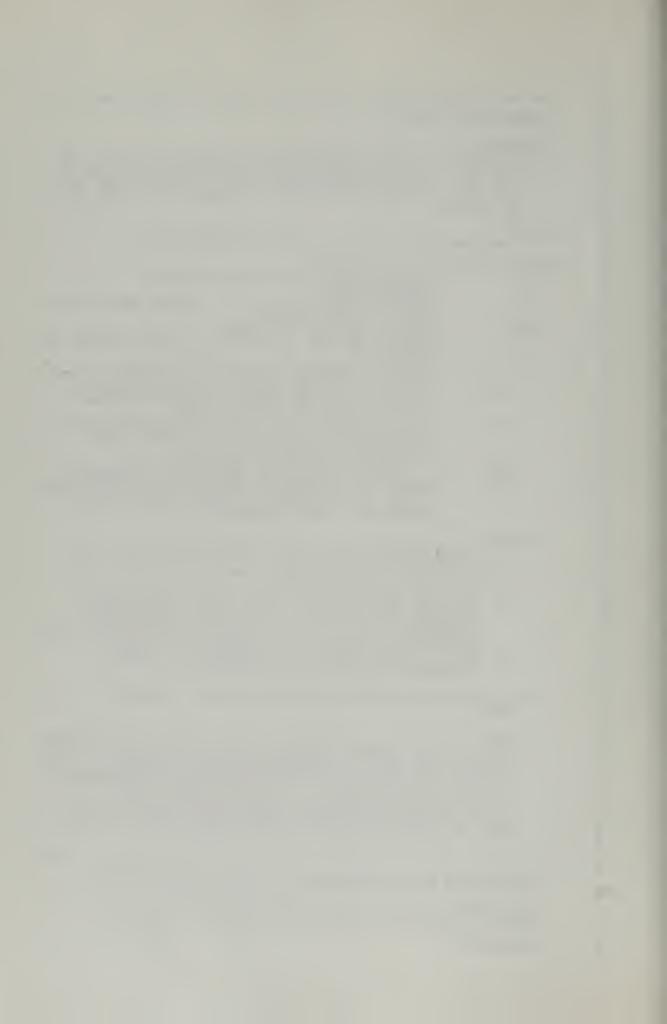
SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED NONE

METHOD HOUSEHOLDER TRANSFORMATIONS ARE USED TO TRANSFORM MATRIX A TO UPPER TRIANGULAR FORM. AFTER HAVING APPLIED THE SAME TRANSFORMATIONS TO MATRIX B, AN APPROXIMATE SOLUTION OF THE PROBLEM IS COMPUTED BY BACK SUBSTITUTION. FOR REFERANCE, SEE GOLUB, G., NUMERICAL METHODS FOR SOLVING LINEAR LEAST SQUARES PROBLEMS, NUMERISCHE MATHEMATIK, VOL. M, ISS.3 (1965), PP.206-216.

SUBROUTINE DLLSQ(A, B, M, N, L, X, IPIV, EPS, IER, AUX)

DIMENSION A(1), B(1), X(1), IPIV(1), AUX(1)
DOUBLE PRECISION A, B, X, AUX, PIV, H, SIG, BETA, TOL

ERROR TEST



```
IF(M-N)30,1,1
CCCC
            GENERATION OF INITIAL VECTOR S(K) (K=1,2,...,N) IN STORAGE LOCATIONS AUX(K) (K=1,2,...,N)
            PIV=0.DO
            IEND=O
DC 4 K=1,N
IPIV(K)=K
           H=0.D0

IST=IEND+1

IEND=IEND+M

D0 2 I=IST, IEND

H=H+A(I)*A(I)

AUX(K)=H

IF(H-PIV)4,4,3
            PIV=H
        3
           KPIV=K
CONTINUE
CCC
             ERROR TEST
             IF(PIV)31,31,5
        DEFINE TOLERANCE FOR CHECKING RANK OF A SIG=DSQRT(PIV) TOL=SIG*ABS(EPS)
            DECOMPOSITION LOOP LY=L*M
            IST=-M
IST=-M
DO 21 K=1,N
IST=IST+M+1
IEND=IST+M-K
I=KPIV-K
             IF(1)8,8,6
            INTERCHANGE K-TH COLUMN OF A WITH KPIV-TH IN CASE KPIV.GT.K.
H=AUX(K)
AUX(K)=AUX(KPIV)
AUX(KPIV)=H
ID=I*M
DO 7 I=IST, IEND
             J = I + ID
        H = A(I)

A(I) = A(J)

A(J) = H
            CCMPUTATION OF PARAMETER SIG IF(K-1)11,11,9
SIG=0.DO
DO 10 I=IST,IEND
SIG=SIG+A(I)*A(I)
SIG=DSQRT(SIG)
         ğ
       10
C
             TEST ON SINGULARITY IF(SIG-TOL)32,32,11
             GENERATE CORRECT SIGN OF PARAMETER SIG
            H=A(IST)
IF(H)12,13,13
SIG=-SIG
       11
       12
             SAVE INTERCHANGE INFORMATION IPIV(KPIV)=IPIV(K) IPIV(K)=KPIV
CCC
             GENERATION OF VECTOR UK IN K-TH COLUMN OF MATRIX A AND OF PARAMETER BETA
             BETA=H+SIG
```



```
A(IST)=BETA
             BETA=1.DU/(SIG*BETA)
J=N+K
AUX(J)=-SIG
IF(K-N)14,19,19
             TRANSFORMATION OF MATRIX A PIV=0.00
       14
              ID=C
             JST=K+1
KPIV=JST
DO 18 J=JST,N
ID=ID+M
             H=0.D0
D0 15 I=IST, IEND
II=I+ID
H=H+A(I)*A(II)
             H=BETA*H

DO 16 I=IST, IEND

II=I+ID

A(II)=A(II)-A(I)*H
CC
             UPDATING OF ELEMENT S(J) STORED IN LOCATION AUX(J) II=IST+ID
H=AUX(J)-A(II)*A(II)
AUX(J)=H
IF(H-PIV)18,18,17
PIV=H
       17
             KPIV=J
CONTINUE
       18
              TRANSFORMATION OF RIGHT HAND SIDE MATRIX B
              DO 21 J=K,LM,M
H=0.DO
       19
              IEND=J+M-K
II=IST
DO 20 I=J,IEND
              H=H+A(II)*B(I)
II=II+1
       20
              H=BETA*H
II=IST
DO 21 I=J, IEND
              B(I)=B(I)-A(II)*H
              II = II + 1
       21
              END OF DECOMPOSITION LOOP
CCCC
              BACK SUBSTITUTION AND BACK INTERCHANGE
              IER=O
I=N
              LN=L*N
              PIV=1.DO/AUX(2*N)

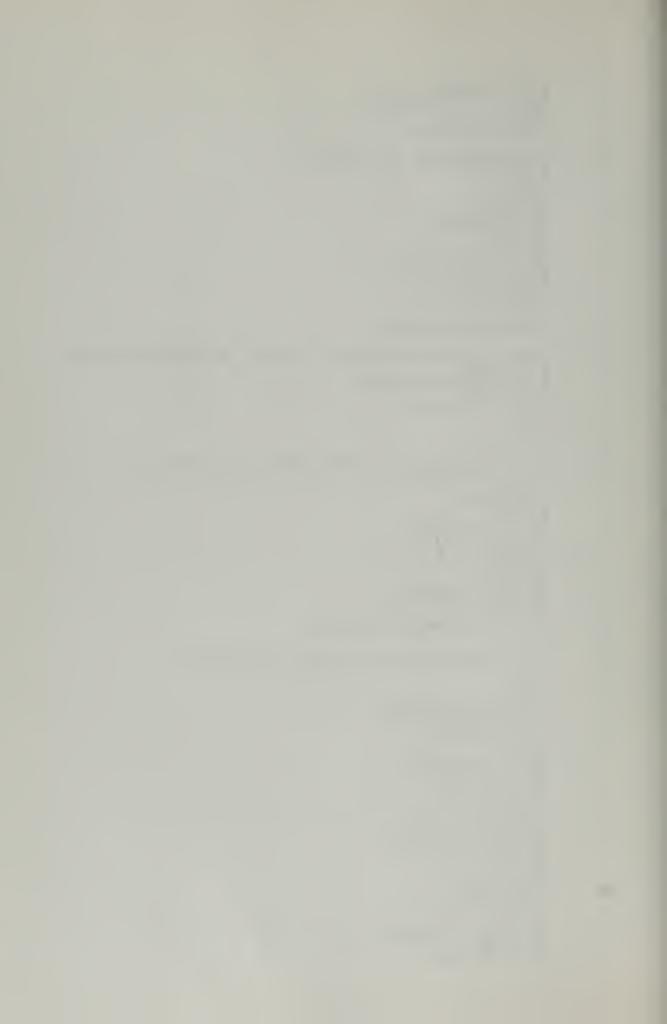
DO 22 K=N,LN,N

X(K)=PIV*B(I)

I=I+M

IF(N-1)26,26,23

JST=(N-1)*M+N
       22
             JST=(N-1)*M+N
DO 25 J=2,N
JST=JST-M-1
K=N+N+1-J
PIV=1.DO/AUX(K)
KST=K-N
ID=IPIV(KST)-KST
IST=2-J
DO 25 K=1,L
H=B(KST)
IST=IST+N
IEND=IST+N
IEND=IST+J-2
II=JST
DO 24 I=IST,IEND
II=II+M
H=H-A(II)*X(I)
       23
              H=H-A(II)*X(I)
```



```
I=IST-1
          II=I+ID
X(I)=X(II)
X(II)=PIV*H
KST=KST+M
CCC
           COMPUTATION OF LEAST SQUARES IST=N+1 IEND=0
     26
           IEND=0

DO 29 J=1,L

IEND=IEND+M

H=0.D0

IF(M-N)29,29,27

DO 28 I=IST,IEND

H=H+B(I)*B(I)

IST=IST+M

AUX(J)=H

PETURN
      27
28
      29
           RETURN
CC
           ERROR RETURN IN CASE M LESS THAN N IER=-2 RETURN
      30
           ERROR RETURN IN CASE OF ZERO-MATRIX A
           IER=-1
RETURN
      31
CC
           ERROR RETURN IN CASE OF RANK OF MATRIX A LESS THAN N
           IER=K-1
RETURN
END
      32
SUBROUTINE ROUND
           PURPOSE
TO ROUND OFF A NUMBER TO A SPECIFIED NUMBER OF
                 SIGNIFICANT DIGITS
           USAGE
                 CALL ROUND (A, N)
           DESCRIPTION OF PARAMETERS

A - NUMBER TO BE ROUNDED

N - SIGNIFICANT DIGITS TO BE RETAINED
           REMARKS
                 NONE
            SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
                 NONE
            . . . . . . . . . . . . . . . .
           SUBROUTINE ROUND(A,N)
REAL*8 A
X=A
IF (X.EQ.O.O) GO TO 1
SIGN=+1.0
IF (X.LT.O.O) SIGN=-1
L=ALOG10(ABS(X))+1.00
Y=X*10.0**(N-L)
I=Y
Z=I
IF ((Y-Z).GT.O.5) I=I
Z=I
A=SIGN*Z*10.0**(L-N)
            SUBROUTINE ROUND (A, N)
                                     SIGN=-1.0
                 ((Y-Z).GT.0.5) I=I+1
           Ä=SIGN*Z*10.0**(L-N)
CONTINUE
RETURN
```

END



```
SUBROUTINE RTPLSB
       PURPOSE
              TO FIND THE ROOTS, BOTH REAL AND COMPLEX, OF A POLYNOMIAL WITH REAL COEFFICIENTS USING BOTH BAIRSTOW AND NEWTON-RAPHSON METHODS.
       USAGE
               CALL RTPLSB(N,A,U,V,CONV,IER)
       DESCRIPTION OF PARAMETERS
               INPUT
                                       DEGREE OF POLYNOMIAL COEFFICIENT VECTOR OF POLYNOMIAL
               N
               OUTPUT
                                      VECTOR OF REAL PARTS OF ROOTS
VECTOR OF IMAGINARY PARTS OF ROOTS
CONVERGENCE INDICATORS FOR EACH ROOT
ERROR INDICATOR
=0 , N IS WITHIN BOUNDS
=1 , N IS LESS THAN ONE
               ŬV
               CONV
               IER
       REMARKS
(1)
                         FOR PROBLEMS WITH NONMULTIPLE ROOTS, THE APPROXIMATE NUMBER OF SIGNIFICANT DIGITS OF EACH PART OF EACH ROOT WILL APPEAR AS THE EXPONENT OF THE CORRESPONDING ENTRY IN CONVACCURACY MAY BE LESS THAN INDICATED BY CONVIF THERE ARE MULTIPLE ROOTS
               (2)
       SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
               NONE
       SUBROUTINE RTPLSB (N,A,U,V,CONV,IER)
IMPLICIT REAL*8 (A-H), REAL*8 (O-Z)
DIMENSION A(1),U(1),V(1),CONV(1),B(53)
DIMENSION C(53),D(53),E(53),H(53)
       KF=10
        L=25
        IER=0
       NN=N
       K=0
N1=N+1
        IZF=0
       IF(NN) 51,51,52
DO 11 I=1,N1
IF(A(I)) 15,12,15
CONTINUE
52
15
       IZF=1
GO TO 11
CONTINUE
IF (IZF-1) 13,14,13
12
       NN=NN-1
13
       K=K+1
CONTINUE
NP3=NN+3
11
14
       NP3=NN+3
N3=NP3
DO 10 I=1,N3
CCNV(I)=0.
U(I)=0.
V(I)=0.
B(2)=0.0
B(1)=0.0
C(2)=0.0
C(1)=0.0
D(2)=0.0
E(2)=0.0
10
```



```
H(2)=0.0
            DO 101 J=3, NP3
H(J)=A(J+K-2)
CONTINUE
101
           CONTINUE
T=1.0
SK=10.0**KF
IF(H(NP3)) 200,151,
U(NP3)=0.0
V(NP3)=0.
CONV(NP3)=SK
NP3=NP3-1
IF(NP3) 51,51,150
IF(NP3-3) 54,54,201
PS=0.0
QS=0.0
PT=0.0
150
151
                                              200, 151, 200
200
201
           PT=0.0
QT=0.0
S=0.0
REV=1.0
SK=10.0**KF
IF(NP3-4) 51,202,203
R=-H(4)/H(3)
GO TO 500
DO 207 J=3,NP3
IF(H(J))204,207,204
S=S+DLOG(DABS(H(J)))
CONTINUE
FPN1=N+1
S=DEXP(S/FPN1)
202
203
204
207
           S=DEXP(S/FPN1)
D0 208 J=3,NP3
H(J)=H(J)/S
CONTINUE
IF(DABS(H(4)/H(3))-DABS(H(NP3-1)/H(NP3)))250,252,252
208
250
            CONTINUE
            M=(NP3-4)/2 + 3

DO 251 J=3,M

S=H(J)

JJ=NP3-J+3

H(J)=H(JJ)

H(JJ)=S

IF(QS) 253,254,253

CONTINUE

P=PS

D=OS
             T = -T
251
252
253
            Q=QS
GO TO 300
HH2=H(NP3-2)
IF(HH2) 256,255,256
254
            Q=1.0
P=-2.0
GG TO
255
            GO TO 257
Q=H(NP3)/HH2
P=(H(NP3-1)-Q*H(NP3-3))/HH2
256
257
258
300
             IF(NP3-5)258,550,258
            R=0.0
CONTINUE
            DO 490 I=1,L

DO 351 J=3,NP3

B(J)=H(J)-P*B(J-1)-Q*B(J-2)

C(J)=B(J)-P*C(J-1)-Q*C(J-2)

CONTINUE

IF(H(NP3-1))352,400,352
351
            CONTINUE
IF (8(NP3-1)) 353,400,353
AVHB1=DABS(H(NP3-1)/B(NP3-1))
IF(AVHB1-SK)450,354,354
B(NP3)=H(NP3)-Q*B(NP3-2)
IF(B(NP3))401,550,401
AVHB2=DABS(H(NP3)/B(NP3))
352
353
354
400
401
             IF(SK-AVHB2)550,450,450
DO 451 J=3,NP3
             DO 451 J=3, NP3
D(J)=H(J)+R*D(J-1)
450
```



```
E(J)=D(J)+R*E(J-1)
CONTINUE
IF(D(NP3))452,500,452
CONTINUE
AVHD3=DABS(H(NP3)/D(NP3))
IF(SK-AVHD3)500,453,453
CC2=C(NP3-2)
CC3=C(NP3-3)
C(NP3-1)=-P*CC2-Q*CC3
CC1=C(NP3-1)
S=CC2*CC2-CC1*CC3
IF(S)455,454,455
CONTINUE
451
452
453
               IF(S)455,454,455

CONTINUE

P=P-2.00

Q=Q*(Q+1.0)

GO TO 456

P=P+(B(NP3-1)*CC2-B(NP3)*CC3)/S

Q=Q+(-B(NP3-1)*CC1+B(NP3)*CC2)/S

IF(E(NP3-1))458,457,458

R=R-1.0

GO TO 490

R=R-D(NP3)/E(NP3-1)

CONTINUE
454
455
456
457
458
                CONTINUE
PS=PT
QS=QT
PT=P
490
                 QT = Q
                IF(REV)491,492,492

SK=SK/10.0

REV=-REV

GO TO 250

IF(T)501,502,502

R=1.0/R

NP=NP3-3
 491
 492
 500
 501
502
               NP=NP3-3

U(NP)=R

V(NP)=C.0

CONV(NP)=SK

NP3=NP3-1

DO 503 J=3,NP3

H(J)=D(J)

IF(NP3-3)3C0,51,300

IF(T)551,552,552

P=P/0
 503
 550
                 P=P/Q
Q=1.0/Q
PP2=P/2.0
QMPSQ=Q-PP2*PP2
 551
 552
                IF(QMPSQ)554,554,553
NP=NP3-3
U(NP)=-PP2
 553
                U(NP)=-PP2
U(NP-1)=-PP2
S=DSQRT(GMPSQ)
V(NP)=S
V(NP-1)=-S
GO TO 561
S=DSQRT(-GMPSQ)
NP=NP3-3
IF(P)555,556,556
U(NP)=-PP2+S
GO TO 557
 554
  555
                 GO TO 557
                 U(NP)=-PP2-S
U(NP-1)=9/U(NP)
  556
557
                 V(NP)=0.0
V(NP-1)=0.0
CONV(NP)=SK
CONV(NP-1)=SK
  561
                 NP3=NP3-2
DO 558 J=3,NP3
H(J)=B(J)
GO TO 200
   558
                   IER=1
       51
54
                  RETURN
END
```



```
SUBROUTINE ARRAY
```

PURPOSE CONVERT SION OR LINK TH CONVERT DATA ARRAY FROM SINGLE TO DOUBLE DIMEN-SION OR VICE VERSA. THIS SUBROUTINE IS USED TO LINK THE USER PROGRAM WHICH HAS DOUBLE DIMENSIO ARRAYS AND THE SSP SUBROUTINES WHICH OPERATE ON ARRAYS OF DATA IN A VECTOR FASHION. USED TO DIMENSION

USAGE CALL ARRAY (MODE, I, J, N, M, S, D)

OF PARAMETERS
CODE INDICATING TYPE OF CONVERSION
=1 - FROM SINGLE TO DUUBLE PRECISION
NUMBER OF ROWS IN ACTUAL DATA MATRIX
NUMBER OF ROWS IN ACTUAL DATA MATRIX
NUMBER OF ROWS SPECIFIED FOR THE MATRIX
D IN DIMENSION STATEMENT
IF MODE=1, THIS VECTOR IS INPUT WHICH
CONTAINS THE ELEMENTS OF A DATA MATRIX
OF SIZE I BY J. COLUMN I+1 OF DATA
MATRIX FOLLOWS COLUMN I, ETC. IF MODE=2
THIS VECTOR IS OUTPUT REPRESENTING A
DATA MATRIX OF SIZE I BY J CONTAINING
ITS COLUMNS CONSECUTIVELY. THE LEGNTH
OF S IS IJ=!*J.
IF MODE=1, THIS MATRIX OF SIZE N BY M
IS OUTPUT, CONTAINING A DATA MATRIX OF
SIZE I BY J IN THE FIRST I ROWS AND J
COLUMNS. IF MODE=2, THIS N BY M MATRIX
IS INPUT CONTAINING A DATA MATRIX OF
SIZE I BY J IN THE FIRST I ROWS AND J
COLUMNS PARAMETERS
DE INDICATING
- FROM SINGL
- FROM DOUBL DESCRIPTION MODE -0F J S

D COLUMNS

REMARKS
VECTOR S CAN BE IN THE SAME LOCATION AS
D. VECTOR S IS REFERANCED AS A MATRIX I
SSP ROUTINES, SINCE IT CONTAINS A DATA M
THIS ROUTINE CONVERTS ONLY GENERAL DATA
(STORAGE MODE O) MATRIX IN OTHER MATRIX. MATRICES

SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED NONE

SUBROUTINE ARRAY (MODE, I, J, N, M, S, D)

DIMENSION S(1), D(1)REAL*8 S,D

NI = N - I

TEST TYPE OF CONVERSION

IF(MODE-1) 100,100,120

CCNVERT FROM SINGLE TO DOUBLE DIMENSION

I J=I*J+1 NM=N*J+1 DO 110 K=1,J NM=NM-NI 100 DO 110 L=1, IJ=IJ-1 NM=NM-1 D(NM)=S(IJ) L=1, I 110



```
120
          DO 130 K=1,J
DO 125 L=1,I
          IJ=IJ+1
          NM=NM+1
S(IJ)=D(NM)
NM=NM+NI
   125
130
140
          RETURN
C
          END
c
          SUBROUTINE GAUSS3
          PURPOSE
          USAGE
          DESCRIPTION
               N
EPS
A
               KER
               K
          SUBROUTINES
(1) MINV
(2) ARRAY
          REMARKS
```

GO TO 140

```
CONVERT FROM DOUBLE TO SINGLE DIMENSION
0=LI
NM=0
        INVERT A DOUBLE PRECISION MATRIX BY THE GAUSS-
JORDAN METHOD. THIS ROUTINE IS A DOUBLE PRECIS-
ION VERSION OF SSP ROUTINE MINV USING F1-NPGS-
        GAUSS3 (F-63) CALLING SEQUENCE
                                  OF PARAMETERS
                                  ORDER OF MATRIX
DUMMY PARAMETER
TWO DIMENSIONAL
TO BE INVERTED
TWO DIMENSIONAL
                                                                               NOT USED BY GAUSS3
ARRAY CONTAINING MATRIX
                                                                              ARRAY CONTAINING INVERTED
                                  TWO DIMENSIONAL ARRAY CONTAINING INVER'
MATRIX
ERROR FLAG
=1 INDICATES NO ERRORS
=2 INDICATES MATRIX IS SINGULAR
ROW AND CCLUMN DIMENSION OF A AND X IN
USERS PROGRAM
                                  AND FUNCTION SUBPROGRAMS REQUIRED
         ALL FLOATING POINT VARIABLES ARE DOUBLE (REAL*8). IF N IS GREATER THAN 50, THE OF ARRAYS L, M, AND Y MUST BE CHANGED TO GREATER THAN OR EQUAL TO N.
                                                                                                                          PRECISION DIMENSION
SUBROUTINE GAUSS3(N, EPS, A, X, KER, K)
REAL*8 A, X, Y, D
DIMENSION A(1), X(1), L(50), M(50), Y(50, 50)
DIMENSION A(1),X(1),L(50),M
DO 1 I=1,N
DO 1 J=1,N
IND=(I-1)*K+J
Y(I,J)=A(IND)
KER=1
N2=2*N
CALL ARRAY(2,N,N,50,50,Y,Y)
CALL MINV(Y,N,D,L,M)
CALL ARRAY(1,N,N,50,50,Y,Y)
IF(D.EQ.O.) KER=2
DO 2 I=1,N
DO 2 J=1,N
IND=(I-1)*K+J
X(IND)=Y(I,J)
RETURN
END
```



```
\circ
```

SUBROUTINE MINV

CALL MINV(A,N,D,L,M)

USAGE

DESCRIPTION

CCC

```
OF PARAMETERS
INPUT MATRIX, DESTROYED IN COMPUTATION
AND REPLACED BY RESULTANT INVERSE.
ORDER OF MATRIX A
RESULTANT DETERMINANT
WORK VECTOR OF LEGNTH N
WORK VECTOR OF LEGNTH N
                      D
                      LM
              SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
                      NONE
              REMARKS
NONE
              SUBROUTINE MINV(A,N,D,L,M)
DIMENSION A(1),L(1),M(1)
DOUBLE PRECISION A,D,BIGA,HOLD
                      SEARCH FOR LARGEST ELEMENT
              D=1.0
NK=-N
D0 80 K=1,N
               NK=NK+N
              NK=NK+N

L(K)=K

M(K)=K

KK=NK+K

BIGA=A(KK)

DO 20 J=K,N

IZ=N*(J+1)

DO 20 I=K,N

IJ=IZ+I

IF(DABS(BIGA)-DABS(A(IJ))) 15,20,20

BIGA=A(IJ)

L(K)=I

M(K)=J

CONTINUE
               CONTINUE
       20
                       INTERCHANGE ROWS
               J=L(K)
IF(J-K) 35,
KI=K-N
DO 30 I=1,N
KI=KI+N
                                  35,35,25
        25
              HOLD=-A(KI)
JI=KI-K+J
A(KI)=A(JI)
A(JI) =HOLD
CCC
                       INTERCHANGE COLUMNS
               I=M(K)
IF(I-K) 45,45,38
JP=N*(I-1)
DO 40 J=1,N
JK=NK+J
JI=JP+J
        35
        38
```

PURPOSE
INVERT A DOUBLE PRECISION MATRIX BY THE GAUSSJORDAN METHOD.



```
HOLD=-A(JK)
A(JK)=A(JI)
A(JI) =HOLD
       40
0000
                     DIVIDE COLUMN BY MINUS PIVOT (VALUE OF PIVOT
                     ELEMENT IS CONTAINED IN BIGA)
             IF(BIGA) 48,46,48
D=0.0
RETURN
DO 55 I=1,N
IF(I-K) 50,55,50
       45
       46
       48
              IK=NK+I
              A(IK)=A(IK)/(-BIGA)
CONTINUE
       55
                     REDUCE MATRIX
              DO 65 I=1,N
IK=NK+I
              HOLD=A(IK)
              IJ=I-N
       1J=1-N

DO 65 J=1,N

IJ=IJ+N

IF(I-K) 60,65,60

60 IF(J-K) 62,65,62

62 KJ=IJ-I+K

A(IJ)=HOLD*A(KJ)+A(IJ)
       65 CONTINUE
                     DIVIDE ROW BY PIVOT
              KJ=K-N

DO 75 J=1,N

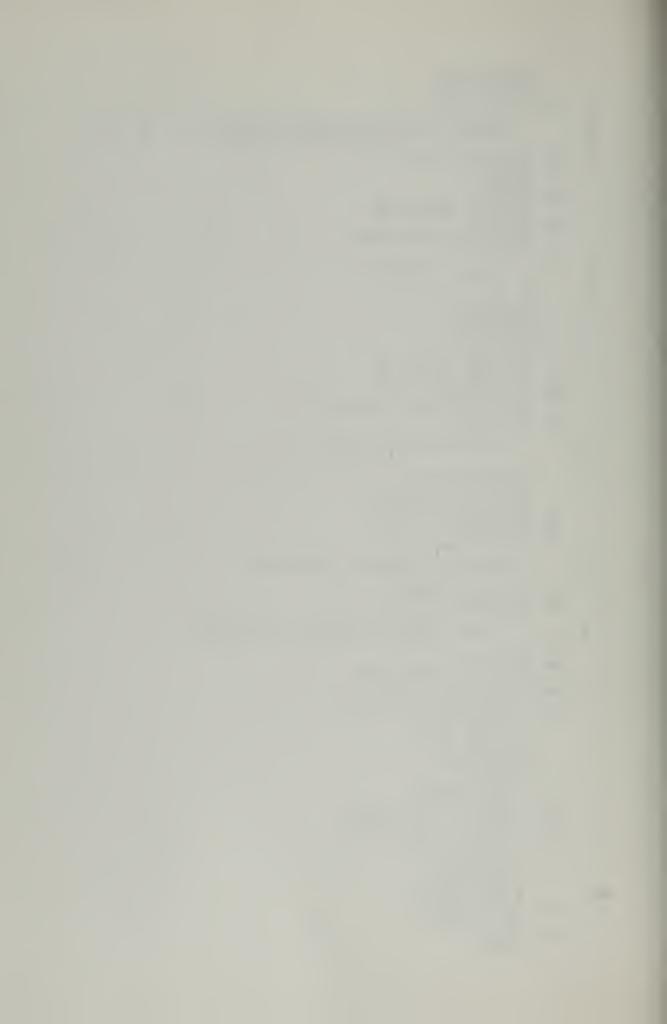
KJ=KJ+N

IF(J-K) 70.75.70

A(KJ)=A(KJ)/BIGA

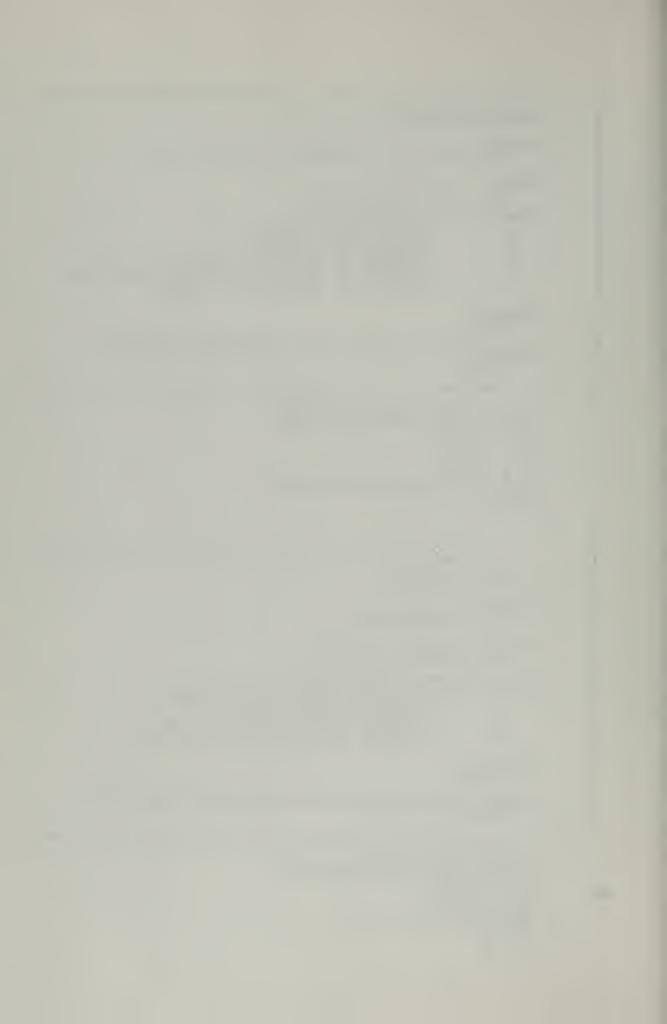
CONTINUE

D=D*BIGA
000
                     REPLACE PIVOT BY RECIPROCAL
        A(KK)=1.0/BIGA
80 CONTINUE
                      FINAL ROW AND COLUMN INTERCHANGE
              K=N
K=(K-1)
IF(K) 150,150,105
I=L(K)
IF(I-K) 120,120,108
JQ=N*(K-1)
JR=N*(I-1)
DO 110 J=1,N
JK=JQ+J
HOLD=A(JK)
JI=JR+J
A(JK)=-A(JI)
A(JI)=HOLD
J=M(K)
IF(J-K) 100,100,125
KI=K-N
DO 130 I=1,N
KI=KI+N
HOLD=A(KI)
               K=N
     100
     105
      108
      110
120
      125
               HOLD=A(KI)
JI=KI-K+J
A(KI)=-A(JI)
A(JI) =HOLD
GO TO 100
      130
               RETURN
END
      150
```



C

```
SUBROUTINE PROD
PURPOSE
TO COMPUTE THE PRODUCT OF TWO MATRICES
USAGE
      CALL PROD (A, B, C, N, M, L)
                         OF PARAMETERS
FIRST INPUT MATRIX
SECOND INPUT MATRIX
PRODUCT OF A AND B
NUMBER OF ROWS IN A
NUMBER OF COLUMNS I
NUMBER OF COLUMNS I
DESCRIPTION
      ABCN
                                                                IN
IN
                                                                     AND C
                                                                       A AND
B AND
      M
                                                                                   ROWS
REMARKS
NONE
SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
      NONE
SUBROUTINE PROD(A,B,C,N,M,L)
REAL*8 A(9,9),B(9,9),C(9,9)
DO 1 I=1,N
DO 1 J=1,L
C(I,J)=0.00
DO 1 K=1,M
C(I,J)=C(I,J)+A(I,K)*B(K,J)
RETURN
END
SUBROUTINE SUMM
PURPOSE ADD TWO MATRICES
USAGE
CALL SUMM(A,B,C,M,N)
                         OF PARAMETERS
NAME OF FIRST INPONAME OF SECOND INTO NAME OF OUTPUT MAY NUMBER OF ROWS IN NUMBER OF COLUMNS
DESCRIPTION
                                                         INPUT MATRIX
       ABCM
                                                          INPUT MATRÎX
MATRIX
IN A,B, AND C
INS IN A,B, AND C
       N
REMARKS
NONE
SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
SUBROUTINE SUMM(A,B,C,M,N)
REAL*8 A(9,9),B(9,9),C(9,9)
DO 1 I=1,M
DO 1 J=1,N
C(I,J)=A(I,J)+B(I,J)
RETURN
 END
```



```
SUBROUTINE DIFF
   PURPOSE
SUBTRACT ONE MATRIX FROM ANOTHER
   USAGE
CALL
                   DIFF(A, B, C, M, N)
                            OF PARAMETERS
FIRST INPUT MATRIX
SECOND INPUT MATRIX
OUTPUT MATRIX EQUALS A - B
NUMBER OF ROWS IN A,B, AND C
NUMBER OF COLUMNS IN A,B, AND C
   DESCRIPTION
         ABCM
         N
   REMARKS
NONE
   SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
          NONE
   SUBROUTINE DIFF(A,B,C,M,N)
REAL*8 A(9,9),B(9,9),C(9,9)
DO 1 I=1,M
DO 1 J=1,N
C(I,J)=A(I,J)-B(I,J)
RETURN
END
    SUBROUTINE EXPAND
   PURPOSE
TO COMPUTE THE REAL COEFFICIENTS OF AN N-TH
DEGREE POLYNOMIAL GIVEN N COMPLEX ROOTS
   USAGE
CALL
                   EXPAND(N,R,A)
                            OF PARAMETERS
DEGREE OF POLYNOMIAL
VECTOR OF COMPLEX ROOTS
COEFFICIENT VECTOR
    DESCRIPTION
         NR
          A
    REMARKS
          NONE
    SUBROUTINES AND FUNCTION SUBPROGRAMS REQUIRED
          NONE
   SUBROUTINE EXPAND(N,R,A)
REAL*8 A(10)
CCMPLEX*16 R(9),Q(9)
IF (N-1) 1,2,3
A(1)=1.00
1
   A(1)=1.00

RETURN

A(1)=-REAL(R(1))

A(2)=1.00

RETURN

Q(1)=-R(1)

Q(2)=1.00

D0 5 J=2,N
2
3
```



```
Q(J+1)=1.00

JJ=J-1

DO 4 I=1,JJ

K=J-I

4 Q(K+1)=Q(K)-Q(K+1)*R(J)

5 Q(1)=-Q(1)*R(J)

6 N1=N+1

DO 7 L=1,N1

7 A(L)=REAL(Q(L))

RETURN

END
```



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ORIGINATING ACTIVITY (Corporate author)

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UNCLASSIFIED

25 GROUP

3 REPORT TITLE

IDENTIFICATION OF SYSTEM DYNAMICS USING MULTIPLE INTEGRATIONS

4 DESCRIPTIVE NOTES (Type of report and inclusive dates)

Master's Thesis; June 1971

5. AUTHOR(S) (First name, middle initial, lest name)

William R. Hansell

June 1971	74. TOTAL NO. OF PAGES	7b. NO. OF REFS
b. PROJECT NO.	98. ORIGINATOR'S REPORT N	UMBER(3)
c. d.	9b. OTHER REPORT NO(5) (An this report)	y other numbers that may be assigned

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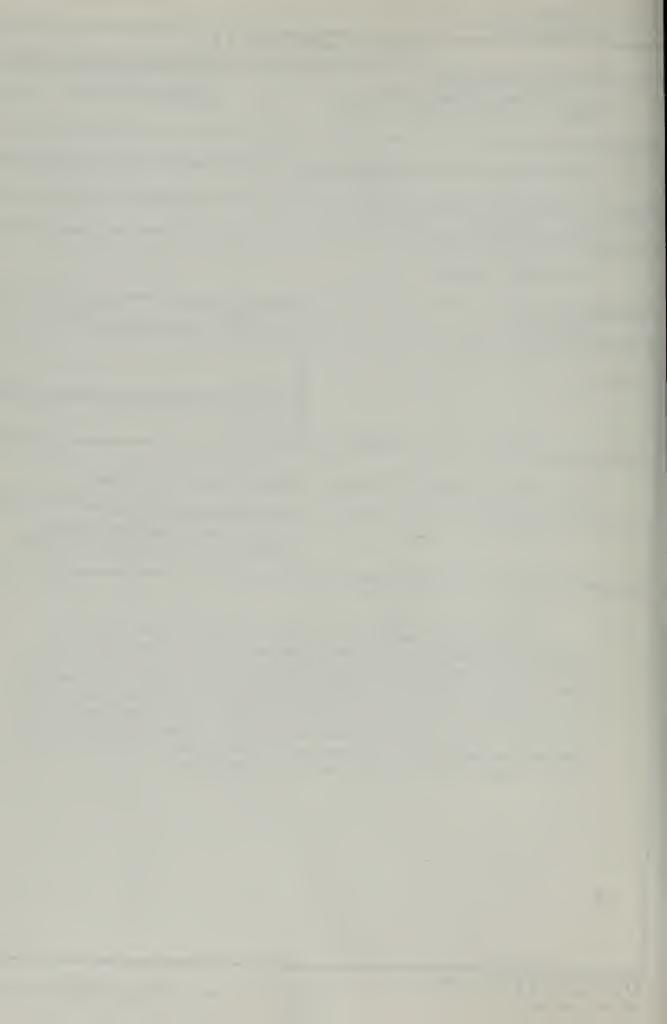
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13 ABSTRACT

A practical method for identifying linear time invariant systems on the basis of arbitrary input-output records is reviewed and extended to handle the case where the system is not initially in the zero state. The method is implemented using a digital computer program composed of a numerical integration subroutine and a subroutine for solving overdetermined sets of linear algebraic equations. Several examples are presented to demonstrate the accuracy and present capabilities of the procedure.



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